Keeping up with the Curve: Learning, Evolving Beliefs and the Anchoring of Expectations *

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Abstract

I explain the behavior of long-run inflation expectations in the post-war U.S. by estimating a forward-looking model in which private agents learn about macroeconomic fundamentals and the policymakers' stabilization preferences in real-time using structural reasoning: recognizing the feedback from their expectations under imperfect knowledge to observed aggregate dynamics. Monetary policy is conducted optimally but private agents suspect that the policymakers' stabilization goals are evolving, which they learn about by interpreting policy decisions. This gives rise to a nonlinear filtering problem. The model provides a novel, information-theoretic explanation of how systematic monetary policy shapes expectations when agents are learning. By demonstrating an emphasis on minimizing unemployment fluctuations through its policy, the Fed has attenuated the sensitivity of long-run inflation expectations to monetary policy surprises, thereby stabilizing them. However, this has made short-run expectations more susceptible to supply shocks perceived by the agents. The model offers a new explanation for the post-pandemic inflation surge in the U.S and the "costless disinflation" that followed, shedding light on why it differed markedly from the crises of the 1970s and the 80s.

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1 Introduction

Inflation surged in the United States in early 2021, reaching a forty-year high of 9.1 percent by June 2022 and catching both the private sector and policymakers off guard. Remarkably, for nearly a year, inflation continued to rise while the Federal Reserve maintained the policy rate at the effective lower bound, raising it only in March 2022, prompting widespread concern that the Fed had fallen “behind the curve”. The episode drew inevitable comparisons to the Great Inflation of the 1970s and early 1980s. Both periods were marked by sharp increases in oil prices and significant supply disruptions, yet they produced strikingly different macroeconomic outcomes: a protracted inflationary spiral culminating in a painful disinflation in the early 1980s versus the relatively swift “soft landing” that has characterized the U.S. economy’s recovery from the pandemic shock.

This paper provides a novel structural framework that explains how a shift in the private agents’ perceptions regarding the Fed’s policy objectives, as a result of them learning from the systematic conduct of policy over the post-war period has caused long-run inflation expectations to become stable, enabling the soft landing. The mechanism involves agents continually updating their beliefs about the parameters and shocks describing the latent state of the economy, based on aggregate economic indicators as well as about the Fed policymakers’ dual mandate, based on policy decisions. A perceived shift in focus towards real stabilization has diluted the degree to which interest rates are informative about shifts in the Fed policymakers’ inflation target. This paper demonstrates how this learning channel has led to long-run inflation expectations becoming five times less responsive to monetary policy surprises in the recent period as compared to during the late 1960s - in a trend that has accelerated since the Global Financial Crisis. I describe how at the same time, this channel has caused agents’ short-run inflation expectations to become more responsive to persistent supply shocks while their long-run inflation expectations have become more stable. The estimated model shows how, even as the agents perceived persistent aggregate supply shocks of similar magnitude during the recent surge as they did during the 70s and the 80s, and saw a similarly dovish tilt in Fed policy at the beginning of the crisis, their long-run expectations remained “anchored”.

I develop a model with three key features. First, neither the private agents - households and firms nor central bank policymakers possess full information - their behavior is shaped by their perceptions regarding the fundamental state of the economy; formed based on reasoning about macroeconomic outcomes through the lens of a forward-looking structural model under perpetual learning. Second, monetary policy is conducted optimally under discretion by the central bank policymakers reflecting a dual mandate, weighing between stabilizing inflation around a target and unemployment around its "natural" level.[1] Third, agents do not observe the central bank's policy objectives and suspect that they evolve over time, and hence they continually update their beliefs about them based on interpreting observed policy behavior, taking the form of a nonlinear "inverse problem". I estimate the model using monthly data on inflation, unemployment and the effective federal funds rate over the postwar period in the U.S (1961 to 2024), together with 10-year ahead inflation expectations from the Survey of Professional Forecasters (SPF). To the best of my knowledge, this is the first paper to explain the behavior of monetary policy and the dynamics of short-run and long-run inflation expectations during the postwar period within a unified framework.

In the model, monetary policy does not conform to any instrument-rule but is rather guided by time-varying policy objectives of the Fed policymakers.[2] Their policy decisions *reveal* information regarding the underlying policy objectives. Hence, there is a non-trivial role for a "learning-channel" of monetary policy transmission in this environment through which the conduct of monetary policy affects agents' expectations by affecting their beliefs about future policy behavior. Unanticipated policy behavior is partly attributed to idiosyncratic errors and partly to changes in the policymakers' stabilization objectives: their inflation target and the weight they place on stabilizing the unemployment gap - the deviation of the unemployment rate from the perceived natural level.

Private agents' expectations are shaped by beliefs about both the structural features of the economy and the time-varying objectives of policymakers. Hence, macroeconomic shocks by themselves do not cause agents' expectations to de-anchor so long as policy responds the way they expect. It is the interaction between monetary policy, their perceptions about the state of the macroeconomy as well as their beliefs regarding the incentives driving policymakers' behavior that determine whether expectations are anchored and whether they remain anchored, rather than macroeconomic shocks in isolation.

The model shows that agents' prior beliefs about the policymakers' stabilization preferences shape how they interpret policy behavior. Anchoring is state-dependent. When agents perceive that the central bank places substantial weight on stabilizing unemployment gaps—rather than focusing on maintaining inflation around a target—their beliefs about the target remain more

[1] Time-variation in policymakers' stabilization preferences precludes commitment. The setup here is different from Kydland and Prescott (1977) where a time-invariant, commonly known social welfare function is assumed to exist. Here, policy is based on a legislative dual mandate, open to subjective interpretations by the policymakers.

[2] Instrument-rules such as the Taylor-rule are opaque regarding the trade-offs that policymakers actually face while deliberating about the conduct of monetary policy. It also causes an inconsistent asymmetry regarding how policy behavior is modeled, departing from the forward-looking optimizing framework modeling household and firm behavior. This critique is outlined in Svensson (2003). Nakamura, Riblier and Steinsson (2025) further highlight the deficiencies of Taylor-rule in describing central bank policy.

firmly anchored. In such an environment, agents view interest-rate movements as less informative about changes in the inflation target, rendering long-run inflation expectations less sensitive to policy surprises. At the same time, they also expect a weaker policy response to aggregate supply shocks, since offsetting them would require costly real-side adjustments. Hence their short-run inflation expectations rise by a larger magnitude when they perceive persistent supply shocks in the economy. Thus the goals of stabilizing inflation in the short-run against supply shocks and stabilizing long-run inflation expectations are in tension when agents are learning. This learning mechanism is key to understand why the two most prominent inflation crises of postwar U.S. produced such different macroeconomic outcomes.

The estimates imply a profound shift in agents' perceptions of the Fed's policy priorities over the postwar era. The perceived weight on minimizing the unemployment gap. nearly negligible during the 1960s, has become more than twice as large as its inflation stabilization objective, signaling a regime in which real-side stabilization has become central to systematic policy. Beginning in the late 1960s, a sustained period of expansionary monetary policy conveyed that the inflation target had begun to drift upwards. Even as supply pressures mounted in the form of two great oil shocks in the 70s and real activity softened, policy maintained an over-accommodative stance. This behavior led to the agents sharply revising their beliefs about the inflation target upwards, resulting in the de-anchoring of long-run inflation expectations. Short-run and long-run inflation expectations rose in unison.

Over the recent years, particularly after the Global Financial Crisis, the agents have perceived the Fed to put a much greater emphasis on stabilizing the unemployment gap. This development has had important implications for the recent inflation surge. Even as the Fed demonstrated an unwillingness to respond to the inflation crisis, long-run inflation expectations remained remarkably stable: agents were slower to interpret the delayed policy response as evidence of a shift in the Fed's target. They rationalized the behavior as an increase in the policymakers' concern against creating unemployment gap volatility. This however, magnified the *pass through* from perceived supply shocks to short-run inflation expectations since they perceived little incentive from policymakers to dampen the effects through adjustments on the real side. The mechanism explains the growing discord between the behavior of short- and long-horizon inflation expectations, also shedding light on how stabilizing short-run and long-run inflation expectations can be at odds when agents are learning using structural reasoning.

Much of the existing literature on learning has focused on *adaptive learning*, where agents form expectations using simple econometric models conditioning on the observed history of macroeconomic data. Agents do not internalize the general equilibrium from their beliefs to macroeconomic outcomes, ignoring the forward-looking nature of aggregate economic dynamics that would arise from their optimizing behavior. This paper models *structural* learning (see Williams (2003)): agents form expectations based on a structural forward-looking model. While their knowledge is imperfect, their beliefs remain model-consistent and rational at all times.

The constraints posed by the information structure of the economy can make learning about certain structural features exceptionally slow. This is particularly relevant for slope of the expectations-augmented Phillips curve, a key variable acting as the structural link between aggregate inflation and economic slack. Only aggregate demand shocks deliver information about this parameter and hence, agents update their beliefs about the slope primarily around demand-driven recessions.

Related literature: The New-Keynesian model emphasizes the importance of expectations for understanding the structure of aggregate dynamics and individual economic behavior. The literature has highlighted the importance of explicitly accounting for agents' expectations-formation process, using survey data for understanding macroeconomic phenomena (see Coibion et al. (2018), Adam and Padula (2011), Milani (2023), Del-Negro and Eusepi (2011)). This paper models how agents form expectations in real-time, employing survey data and New-Keynesian principles to get a disciplined perspective on how their evolving beliefs have shaped the postwar U.S. economy.

This paper is closely related to the literature examining how private agents' and policymakers' learning contributed to the Great Inflation of the 1970s and 1980s (e.g., Bullard and Eusepi (2005); Primiceri (2006); Sargent, Williams and Zha (2006),Orphanides and Williams (2005)). More broadly, it connects to the literature on how the private sector learns from monetary policy. For instance, Melosi (2016) examines the signaling role of policy in a DSGE framework, where monetary policy is informative about fundamental shocks. By contrast, in this paper, the only source of information asymmetry arises from the private sector's inability to observe the central bank's possibly time-varying stabilization preferences.[3] Other empirical papers has explored how agents infer policy targets from macroeconomic data (e.g., Kozicki and Tinsley (2005); Erceg and Levin (2003); Orphanides and Williams (2006)). However, much of this literature relies on adaptive learning, in which agents form beliefs using reduced-form forecasting rules based on historical correlations. In contrast, the learning behavior in this paper is grounded in a structural, forward-looking framework rooted in New Keynesian principles, where agents

[3]Hence this paper is also broadly related to the literature examining time-variation in the Fed's policy targets such as Ireland (2007), Cogley, Primiceri and Sargent (2010), Del-Negro and Eusepi (2011), and Lakdawala (2016). However, the focus in this paper is on private agents' beliefs about the Fed's objectives rather than the true policy objectives themselves.

understand and internalize the effects of their expectations on macroeconomic outcomes.

The causes of the Great Inflation and the subsequent Great Moderation have been the subject of long-standing debate. One school attributes the volatility of the 1970s–80s era to "bad luck": a sequence of large, adverse non-policy shocks (e.g., Sims and Zha (2006); Bernanke and Mihov (1998)). Under this view, the structure of the economy and policy rules remained largely stable; it was the volatility of exogenous shocks that changed. This view implies that monetary policy made little fundamental progress—if similar shocks were to recur, so would similar outcomes. However, I find that systematic monetary policy has shaped agents' expectations in important ways, making their long-run inflation expectations more resilient.

An alternative view emphasizes policy errors. Clarida, Gali and Gertler (2000), Taylor (1993) and Lubik and Schorfheide (2004) argue that the shift to systematic anti-inflation policy by the Fed under the chairmanship of Paul Volcker was central to the subsequent stability. DeLong (1996) and Romer and Romer (2002) provide narrative evidence that policymakers in the 1970s were misled by a perceived long-run tradeoff between inflation and unemployment. Some papers study whether these policy mis-steps by the Fed were a result of "wrong" incentives. For example Christiano and Gust (2000) contend that the Fed fell into an "expectations trap", lacking the incentives to take decisive action to rein in inflation expectations during the 70s. Bianchi and Ilut (2017) instead find that inflation during this period was fiscally led: Fed policy accommodated expansionary fiscal policy by remaining passive. In this paper, the focus is on whether private agents perceived policy surprises and to what extent these affected their beliefs rather than an ex-post evaluation of policy behavior.

A related strand of literature has studied the state-dependent nature of anchoring. Carvalho, Eusepi, Moench and Preston (2023) and Gati (2023) describe anchoring in terms of how agents weight recent inflation data when updating their beliefs about the long-run mean of inflation based on past forecast errors (forecast-switching).[4] In this paper, anchoring directly results from agents learning about the central bank's stabilization preferences based on the policy response to inflation rather than the behavior of inflation itself.

This paper contributes to the debate by offering a framework that disentangles these explanations and sheds light on how each of these contributed to shaping the two prominent inflationary episodes in the post-war period in the U.S.

Outline: In Section 2, I motivate the agents' perceived structural model describing the economy

[4] A related line of research (King and Lu (2022) and Debortoli and Lakdawala (2016)) models anchoring in the context of the ability of the policymakers to commit, as described in Barro and Gordon (1983).

as perceived by private agents which they use to reason about the macroeconomic environment that they face. In Section 3, I describe how agents recursively update their beliefs regarding the policy-invariant structural variables in their model conditional on macroeconomic observations. In Section 4, I describe the conduct of monetary policy from the perspective of the agents and describe how they update beliefs regarding the policymakers' stabilization preferences conditional on observed nominal interest rates. In Section 5, I describe the estimation methodology and the data. In Section 6, I discuss how the agents' beliefs have evolved during the postwar period in the U.S. under learning followed by concluding remarks.

2 A forward looking model under imperfect knowledge

Here I begin by describing the information structure of the economy, defining the set of signals that agents observe at any time, based on which they form their beliefs and make decisions. I then describe a model of firm and household behavior representing the agents' economic reasoning. Finally, I derive the agents' perceived laws of motion describing the aggregate economic dynamics, which they use to interpret observed macroeconomic outcomes and update their beliefs.

2.1 Information structure and learning

Time is discrete ($t = 0, 1, 2, 3, \dots$). The only signals available to the agents are the realizations of the aggregate inflation rate, unemployment rate and the nominal interest rate. At the beginning of each period t, firms and households make their decisions conditional on the history of signals observed until then. The period-t inflation π_t and unemployment rate u_t then realize and are observed by the agents. The central bank then sets the interest rate i_{t+1} in anticipation of future outcomes. The agents interpret the policy decision and form beliefs regarding the policymakers' stabilization preferences.

The entire history of aggregate signals observed by agents up to (and including) period t_0 is denoted by:

$$\mathcal{H}^{t_0} = \{i_{t+1}\}_{t=-\infty}^{t_0} \cup \{z_t\}_{t=-\infty}^{t_0}$$

where $z_t = [\pi_t, u_t]'$ is the vector of realizations of inflation and unemployment, and i_{t+1} denotes the interest-rate policy set in period-t.

2.2 The agents' perceived model of aggregate dynamics

Following New Keynesian principles, I derive the log-linearized equations describing the structural dynamics of the economy from the perspective of the agents, based on a model of optimizing behavior by firms and households under imperfect knowledge. I assume that all private agents have common beliefs, formed conditional on the same data and model.[5] I denote the private agents' expectations, formed conditional on history $\mathcal{H}^t$ as E_t^P.

2.2.1 Aggregate demand: Households' consumption smoothing problem

Consider the problem of a representative household, deriving utility from consumption of an aggregate good and saving in nominal bonds every period conditional on their beliefs formed based on the history of signals observed in the past. In period-t, the household solves:

$$\max_{\{C_{t+k}\}_{k=0}^{\infty}} \; E^P\left[\sum_{k=0}^{\infty} \beta^k \frac{C_{t+k}^{1-\sigma}}{1-\sigma} \Big| \mathcal{H}^{t-1}\right] \tag{1}$$

where C_t is consumption, β is a discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and E^P denotes the representative household's expectations, formed conditional on its model.

The household's budget constraint in nominal terms is:

$$P_t C_t + B_t = (1 + i_{t-1}) B_{t-1} + W_t + T_t + \Pi_t \tag{2}$$

where P_t is the aggregate price level, B_t are nominal bond holdings, i_{t-1} is the nominal interest rate in period $t-1$, W_t denotes their nominal wage-income, T_t denote lump-sum government transfers and Π_t denotes profits from the ownership of firms.

In this setting, the only dynamic decision that the representative household makes is the savings decision, which leads to the following Euler equation:

$$C_t^{-\sigma} = \beta E^P \left[C_{t+1}^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \Big| \mathcal{H}^{t-1} \right] \tag{3}$$

where $1 + \pi_{t+1} = P_{t+1}/P_t$ is the aggregate inflation rate.

[5] I abstract from heterogeneity in firms and households' belief-formation.

Since there is no investment or government expenditure in the model, all output is consumed every period $Y_t = C_t$, log-linearizing around the steady state, we get the aggregate-demand IS-curve:

$$\hat{y}_t = E^P_{t-1}[\hat{y}_{t+1}] - \frac{1}{\sigma}(i_t - E^P_{t-1}[\pi_{t+1}] - \rho) \tag{4}$$

where $\hat{y}_t$ denotes the log-deviation of output from its steady-state. $\rho = -\log\beta$. Defining the *ex-ante* real interest rate as $r_t = i_t - E^P_{t-1}[\pi_{t+1}]$, the IS curve may be more compactly written as:

$$\hat{y}_t = E^P_{t-1}[\hat{y}_{t+1}] - \frac{1}{\sigma}(r_t - \rho) \tag{5}$$

At some hypothetical real interest rate, output would continue along the trend path, y^n_t (or the "natural" level of output) absent cyclical fluctuations. We call this the natural rate of interest and denote it by r^n_t. Log-deviations in the natural level of output $\hat{y}^n_t$ follow:

$$\hat{y}^n_t = E^P_{t-1}[\hat{y}^n_{t+1}] - \frac{1}{\sigma}(r^n_t - \rho) \tag{6}$$

Subtracting Equation 6 from Equation 5 yields:

$$\hat{y}_t - \hat{y}^n_t = E^P_{t-1}[\hat{y}_{t+1} - \hat{y}^n_{t+1}] - \frac{1}{\sigma}(r_t - r^n_t) \tag{7}$$

Denoting $\hat{y}_t - \hat{y}^n_t$ as output gap $\tilde{y}_t$, we may re-write the dynamic IS curve in terms of output gap as:

$$\tilde{y}_t = E^P_{t-1}[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - E^P_{t-1}[\pi_{t+1}] - r^n_t) \tag{8}$$

2.2.2 Aggregate supply: Firms' pricing problem

Firm price-setting under dispersed information Every period t, all firms automatically adjust their prices by the perceived trend inflation rate conditional on their $t-1$ information set, given by π^{*P}_{t-1}.[6] The perceived level of trend inflation does not enter firms' pricing problem since all firms have the same beliefs and adjust to the same perceived trend rate every period. However, relative price adjustments are subject to Calvo frictions. Only a constant fraction $(1-\theta)$ of firms are able to adjust their price every period (adjustment frequency θ is common knowledge). All firms set prices in order to earn a constant markup over their nominal marginal cost. The steady-state relative price of each firm is unity. For concision, we express their optimal price setting problem in terms of log-deviations from steady-state.

[6]This assumption permits the model to be log-linearized around the zero inflation steady state despite non-zero trend inflation. Inflation here refers to inflation beyond this "normal" level.

In the absence of Calvo frictions, log-deviation in firm j's optimal price is equal to the sum of log-deviations in the aggregate price-level, denoted as $\hat{p}_t$ and log-deviations in their real marginal cost, $mc_t(j)$.

$$\hat{p}_t^*(j) = \hat{p}_t + \hat{mc}_t(j)$$

Under Calvo frictions, firms cannot set prices flexibly, so that each firm must set prices in order to receive a constant markup *in expectation*. Their optimal price is given by:

$$\hat{p}_t^*(j) = (1 - \beta\theta)E^j\Big[\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p}_{t+k} + \hat{mc}_{t+k}(j))|\mathcal{H}^{t-1}\Big] \tag{9}$$

where $E^j[\cdot]$ denotes the expectation operator conditional on firm j's information set. The firms' information set is the same as that of the representative households, and is denoted by $\mathcal{H}^{t-1}$

Additionally, each firm observes its own real marginal cost $\hat{mc}_t(j)$ at the beginning of the period. $\hat{mc}_t(j)$ is composed of an economy-wide component $\hat{mc}_t$ plus an idiosyncratic firm-specific *i.i.d* (across firms and across time) component, $\varepsilon_t(j)$ with *large* variance.[7]

$$\hat{mc}_t(j) = \hat{mc}_t + \varepsilon_t(j) \quad \varepsilon_t(j) \sim N(0, \sigma_\varepsilon^2)\,\forall\, j \in (0,1) \tag{10}$$

Under knowledge that it can only reset with probability $1 - \theta$ any period in the future would, the optimal price is given by:

$$\hat{p}_t^*(j) = (1 - \beta\theta)E^j\Big[\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p}_{t+k} + \hat{mc}_{t+k}(j))|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)\Big] \tag{11}$$

Dispersed information assumption: The firm is atomistic, its own marginal cost reveals negligible information about innovations to the economy-wide real marginal costs or changes in the aggregate price level p_t. Thus, their expectations regarding these variables are unchanged upon observing $\hat{mc}_t(j)$ and common across all firms, denoted as E_{t-1}^P. Hence:

$$E^j[\hat{p}_t + \hat{mc}_t(j)|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)] = E_{t-1}^P\hat{p}_t + \hat{mc}_t(j) \tag{12}$$

$$E^j[\hat{p}_{t+k} + \hat{mc}_{t+k}|\mathcal{H}^{t-1} \cup \hat{mc}_t(j)] \approx E_{t-1}^P\hat{p}_{t+k} + E_{t-1}^P\hat{mc}_{t+k}\,\forall\, k \geq 0 \tag{13}$$

[7]This assumption is motivated by the observation that firm-specific price changes are much more volatile as compared to fluctuations in the aggregate price-level. This modeling framework is based on Nimark (2008).

Log-deviations in firm j's optimal reset price may be written as:

$$\hat{p}_t^*(j) = (1-\beta\theta)(E_{t-1}^P \hat{p}_t + \hat{mc}_t(j)) + \beta\theta E_{t-1}^P \hat{p}_{t+1}^* \tag{14}$$

All firms face the same price-setting problem, just observe a different marginal cost signal. Aggregating across firms yields:

$$\hat{p}_t^* = (1-\beta\theta)(E_{t-1}^P \hat{p}_t + \hat{mc}_t) + \beta\theta E_{t-1}^P \hat{p}_{t+1}^* \tag{15}$$

Under a Calvo-probability of price adjustment of $(1-\theta)$, evolution of the aggregate price index satisfies:

$$\hat{p}_t = \theta\hat{p}_{t-1} + (1-\theta)\hat{p}_t^* \tag{16}$$

Combining Equation 15 and Equation 16 yields the aggregate supply condition in terms of the aggregate inflation rate $\pi_t = \hat{p}_t - \hat{p}_{t-1}$:

$$\pi_t = \beta E_{t-1}^P \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{mc}_t + \frac{(1-\theta)^2(1-\beta\theta)}{\theta}(E_{t-1}^P \hat{mc}_t - \hat{mc}_t) \tag{17}$$

The expectational errors manifest due to the fact that the economy-wide real marginal cost is not perfectly forecast-able. If the firms' private signal revealed $\hat{mc}_t$ perfectly, we would go back to the standard New-Keynesian Phillips curve formulation and the expectational error term goes away. Also, it can be seen that the smaller the degree of price-stickiness (θ), the larger the contribution of these errors to actual inflation.

Pro-cyclicality of marginal cost The economy-wide real marginal cost mc_t is unobservable. Agents can only infer it based on aggregate inflation. The structural dynamics driving the economy-wide component of real-marginal costs are unknown, but agents posit a pro-cyclical relationship: when output is above the flex-price level, it puts upward pressure on real marginal costs across all firms. Variation in $\hat{mc}_t$ above and beyond this pro-cyclical channel are attributed to economy-wide supply shocks, s_t. Hence, they perceive the following structural relationship:

$$\hat{mc}_t = \gamma\tilde{y}_t + s_t$$

where s_t denotes the supply shock, assumed to follow a mean-zero stationary process and independent of the unemployment-gap $\tilde{y}_t$. $\gamma > 0$ represents the link between economy-wide real marginal cost and the cyclical fluctuations in unemployment. It is assumed to be constant, but unknown.

The agents' reasoning motivates the following forward-looking Phillips curve:

$$\pi_t = \beta E^P_{t-1}[\pi_{t+1}] + \psi \tilde{y}_t + \xi_t \tag{18}$$

where (i) $E^P_{t-1}[\pi_{t+1}]$ denotes the private-sector's expectation of inflation in period $t+1$ conditional on history of signals $\mathcal{H}^{t-1}$ (ii) ψ represents the sensitivity of inflation to current output-gap. Agents assume ψ is constant but unknown and hence learn about it every period. (iii) ξ_t is a cost-push shock composed of the supply-shock s_t and the expectational error.

The cost-push shock ξ_t captures the component of inflationary pressures arising from factors over and beyond the output gap. Agents allow for the possibility that the cost-push shocks have a persistent component. Hence they decompose the cost-push shock, ξ_t to into a *persistent* AR(1) component denoted by ξ^p_t, with large persistence ρ and a *transitory* i.i.d normal component ξ^{tr}_t.

$$\xi_t = \xi^p_t + \xi^{tr}_t, \quad \xi^{tr}_t \sim iid\ N(0, \sigma^2_{\xi,tr})$$

The persistent component is perceived to evolve as:

$$\xi^p_t = \rho \xi^p_{t-1} + \varepsilon^p_t, \quad \varepsilon^p_t \sim iid\ N(0, \sigma^2_{\xi,p})$$

Only the persistent component ξ^p_t affects future outcomes. Agents must learn about these shocks in real-time in order to form expectations about future macroeconomic prospects. ξ^{tr}_t functions as observation noise.

Natural rate hypothesis: Agents in the model a-priori assume that monetary policy cannot systematically drive real variables once prices and expectations fully adjust. Under the natural rate hypothesis, the output and the real interest rate attain their natural levels in the long-run, beyond the control of monetary policy. This is equivalent to agents assuming that the discount-factor $\beta \to 1$ so that expectations-augmented Phillips curve is vertical in the long-run.[8] Agents reason that in the long run, monetary policy can only sustain non-zero inflation, not output gap.

2.2.3 Measuring real-economic activity

Economic slack, as measured by the *output gap* is an important variable driving aggregate inflation and forms the important structural link between interest-rate policy and aggregate inflation. However, it is not directly observable. It must be inferred based on real time indicators of eco-

[8]Since otherwise, the long-run unemployment-gap must be non-zero from Equation 20.

nomic activity. I assume that the only real indicator in this economy, informing agents about the state of aggregate demand is the unemployment rate. This is due to the unemployment rate being a salient variable, directly part of the dual mandate that most modern central banks in the world, including the Fed follow. For this reason, it is useful to re-frame the analysis in terms of unemployment gaps, defined as the difference between the realized unemployment rate and the one that would prevail under flexible prices, or the "natural" rate of unemployment.

In the model, the agents posit a linear relationship between the output gap and the unemployment gap[9] defined as the difference between the unemployment rate and the natural rate of unemployment (empirically, the "trend" component of unemployment rate): $x_t = u_t - u_t^n$, we have

$$\tilde{y}_t = -\eta x_t$$

where η is referred to as the Okun's law coefficient.

This yields the following dynamic IS-curve representing the law of motion of the unemployment-gap:

$$x_t = E_{t-1}^P x_{t+1} + \tau(i_t - E_{t-1}^P \pi_{t+1} - r_t^n) \tag{19}$$

where $\tau = \frac{1}{\sigma\eta}$ represents the ratio of the intertemporal elasticity of substitution and the Okun's law coefficient. τ is assumed to be constant and known to all agents.

Similarly, the expectations-augmented Phillips curve in terms of unemployment gap becomes:

$$\pi_t = E_{t-1}^P \pi_{t+1} - \kappa x_t + \xi_t \tag{20}$$

where $\kappa = \psi\eta$ is assumed to be a constant that the agents learn about.

3 Learning: Latent structure of the economy

Every period, conditional on the observed unemployment rate u_t, all agents infer the contemporaneous unemployment gap, x_t and update their beliefs about the natural rate of interest r_t^n. Conditional on these, agents update their beliefs about the slope of the Phillips curve κ, and the persistent supply shock ξ_t^p based on the observed aggregate inflation rate π_t. Thus, they inform themselves regarding all the latent structural variables describing economic dynamics in their

[9] This relationship is sometimes referred to as Okun's law. Ball et al. (2017) find that it has been relatively stable across time.

model, every period. The rest of the section discusses the mechanics of their learning behavior.

3.1 Learning and the Anticipated Utility assumption

This paper takes the anticipated utility approach to learning described in Kreps (1998). Under this assumption, agents form "certainty-equivalent" estimates of the relevant state-variables every period and treat their best estimates as if they were constants known with certainty when making decisions. Agents disregard the possibility of updating their current estimates in the future. This assumption is widely used in the literature on learning in macroeconomics (see Evans and Honkapohja (2001)). It allows us to surmount the computational challenges associated with solving the dynamic problem in the full Bayesian setting. Cogley and Sargent (2008) demonstrate how the anticipated utility assumption provides an excellent approximation to Bayesian decision-making in a wide class of problems.

3.2 Inferring unemployment gap x_t

Conditional on the observed unemployment rate u_t, agents update their estimate of the natural rate u_t^n according to the univariate smoothing algorithm:

$$u_t^n = (1 - g_u)u_{t-1}^n + g_u u_t \tag{21}$$

The natural rate is inferred from a backward-looking filter based on the unemployment rate, putting more weight on recent observations. The gain coefficient g_u determines the degree to which shifts in the unemployment rate are attributed to changes in the natural rate of unemployment. g_u is perceived as time-invariant and known by the agents.

From the natural rate hypothesis, $\lim_{h\to\infty} x_{t+h} = 0$. Hence, the unemployment rate is always expected to converge to the natural rate u_t^n in the long-run equilibrium, consistent with the update-rule Equation 21.[10]

[10] Staiger, Stock and Watson (1997) and Orphanides and van Norden (2002) show that such univariate algorithms deliver estimates that are essentially indistinguishable from the ones derived under more sophisticated procedures. Similarly, Kamber, Morley and Wong (2018) demonstrate that economic slack-measures derived from similar univariate procedures can be robust to revisions and are found to be in line with more refined measures of real-time economic activity (such as the Chicago Fed National Economic Activity Index).

Agents' estimate of the contemporaneous unemployment gap is:

$$x_t = u_t - u_t^n$$

I abstract from issues of real-time mis-measurement and data revisions. I do however find that the agents' real-time measure inferred from data lines up quite well with series that are revised retroactively (the CFNAI index as an example, see Appendix E) suggesting that these issues are not of large consequence over the sample period.

3.3 Recursive updating of beliefs

Agents' perceived model of aggregate dynamics: The equations describing the aggregate laws of motion of the economy from the perspective of the private agents are:

$$x_t = E_{t-1}^P[x_{t+1}] + \tau(i_t - E_{t-1}^P[\pi_{t+1}] - r_t^n)$$

$$\pi_t = E_{t-1}^P[\pi_{t+1}] - \kappa x_t + \xi_t^p + \xi_t^{tr}, \quad \xi_t^{tr} \sim iid\, N(0, \sigma_{\xi,tr}^2)$$

$$\xi_t^p = \rho \xi_{t-1}^P + \varepsilon_t^p\ , \quad \varepsilon_t^p \sim iid\, N(0, \sigma_{\xi,p}^2)$$

Conditional on the observed aggregate inflation rate π_t and the unemployment gap x_t inferred from u_t based on the de-trending procedure, agents update their beliefs regarding κ, ξ_t^p and r_t^n recursively every period treating $[\sigma_{\xi,tr}^2, \sigma_{\xi,p}^2, \tau, \rho]$ as known constants.

Learning the natural rate of interest r_t^n: Conditional on the inferred unemployment-gap x_t, the contemporaneous estimate of the natural rate of interest $r_{t|t}^n$ is given by the dynamic IS curve (Equation 19):

$$r_{t|t}^n = \tau^{-1}(E_{t-1}^P x_{t+1} - x_t) + i_t - E_{t-1}^P \pi_{t+1}$$

Further, agents assume that r_t^n follows a Martingale, so that $r_{t+1|t}^n = r_{t|t}^n$

Learning ξ_t^p and κ: Given the unemployment-gap x_t, beliefs about the persistent component of the cost-push shock ($\xi_{t|t}^p$) and the slope of the Phillips curve ($\kappa_{t|t}$) are recursively updated every period based on observed aggregate inflation rate π_t.

Agents face a signal-extraction problem in real time: they cannot immediately distinguish whether an inflation surprise was driven by the unemployment-gap (due to mis-perception of the slope

κ), persistent supply shocks—relevant for dynamics—or transitory supply shocks, irrelevant for dynamics.

The observation equation, relating aggregate inflation to ξ_t^p and κ in the agents' perceived model is given by:

$$\pi_t - E_{t-1}^P \pi_{t+1} = \begin{pmatrix} -x_t & 1 \end{pmatrix} \begin{pmatrix} \kappa \\ \xi_t^p \end{pmatrix} + \xi_t^{tr}, \quad \xi_t^{tr} \sim N(0, \sigma_{\xi,tr}^2).$$

The state-transition equation specifying the agents' prior regarding how the state variables κ and ξ_t^p evolve over time is given by:

$$\begin{pmatrix} \kappa \\ \xi_t^p \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} \kappa \\ \xi_{t-1}^p \end{pmatrix} + \begin{pmatrix} 0 \\ \epsilon_t^p \end{pmatrix}, \qquad \epsilon_t^p \sim N(0, \sigma_{\xi,p}^2)$$

Note that the state-transition equation reflects the agents' prior that the true slope κ is time-invariant.

4 Learning: Policymakers' stabilization preferences

Given their beliefs regarding the latent structural variables - the slope of the Phillips curve $\kappa_{t|t}$, the persistent supply shock $\xi_{t|t}^p$ and the natural rate of interest $r_{t|t}^n$, the agents interpret the period-t policy decision i_{t+1} and learn about the policymakers' stabilization preferences underlying policy behavior. The next subsection describes the policymakers' optimal policy problem under a dual mandate, based on Svensson and Woodford (2004).

4.1 Monetary policy under discretion

The private-agents perceive the central bank as conducting monetary policy optimally under discretion, having the same beliefs about the policy-invariant latent structural variables and taking their expectations as given. The central bank's *dual mandate* loss function under discretion is given by:[11]:

$$\mathcal{L}_t = (\pi_{t+1|t} - \pi_t^*)^2 + \lambda_t x_{t+1|t}^2 \tag{22}$$

where $\pi_{t+1|t}$ and $x_{t+1|t}$ denote the central bank's forecasts of inflation and the unemployment gap for period $t+1$ formed in period-t after the realization of period-t aggregate variables π_t and u_t. Here, π_t^* and λ_t represent the policymakers' period-t stabilization preferences: their inflation target and the relative weight they place on unemployment gap stabilization. Private agents do not observe these and perceive them to be subject to small innovations over time. Their beliefs regarding these parameters conditional on $\mathcal{H}^t$ are denoted by π_t^{*P} and λ_t^P.

The central bank in its period-t decision cycle chooses the one-period ahead unemployment gap plan $x_{t+1|t}$. Private agents' period-t expectations regarding future inflation and unemployment gaps formed prior to observing the policy decision (denoted by $E_t^{P-}\pi_{t+j}$ and $E_t^{P-}x_{t+j}$) are observed by the central bank and taken as given.[12] Every period, the central bank chooses $x_{t+1|t}$, effectively solving a static optimization problem, taking the observed private-sector expectations as given, conditional on their period-t beliefs

$$\pi_{t+1|t} = E_t^{P-}\pi_{t+2} - \kappa_{t|t}x_{t+1|t} + \xi_{t+1|t} \tag{23}$$

The optimal unemployment gap plan $x_{t+1|t}$ is given by the targeting rule:

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t}(\pi_{t+1|t} - \pi_t^*) \tag{24}$$

Equation 24 describes the textbook "lean against the wind" policy solution where the central bank reacts by creating a positive unemployment gap if it expects inflation to be above its target in the following period.

Using Equation 23 and Equation 24, the central bank's one-period ahead inflation and unemployment gap plan must satisfy:

$$\pi_{t+1|t} = \pi_t^* + \frac{\lambda_t}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{25}$$

$$x_{t+1|t} = \frac{\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \xi_{t+1|t} - \pi_t^*) \tag{26}$$

[11] Agents do not perceive the central bank's policy goals as being fundamentally incompatible. There is no stabilization bias.

[12] E_t^{P-} denote expectations formed after the realization of the contemporaneous aggregates $z_t = [\pi_t, u_t]$ but before the period-t policy decision, and are thus conditional on beliefs $\kappa_{t|t}, \xi_{t|t}^p, \pi_{t-1}^{*P}, \lambda_{t-1}^P$.

Equation 25 shows that when $\lambda_t > 0$, the central bank allows inflation to rise above its target when faced with a persistent cost-push shock. Deviations of inflation from the target, arising from private agents' inflation expectations and persistent cost-push shocks are accommodated to an extent. The larger λ_t is, the greater the accommodation, with $\lambda_t \to \infty$ characterized by the scenario where the central bank can no longer push back against private agents' expectations anymore. This represents an "expectations-trap" in the sense of Christiano and Gust (2000) that policymakers fall into where any real-side adjustment is deemed too costly. [13]

4.2 Private agents' expectations in a learning equilibrium

Under a rational expectations equilibrium, private-agents' expectations for all future horizons must be consistent with their structural model of inflation dynamics:

$$E^P[\pi_{t+j}] = E^P[\pi_{t+j+1}] - E^P[\kappa x_{t+j}] + E^P[\xi_{t+j}] \; \forall \; j \geq 1$$

The constraint above yields the following forward looking system of equations that their expectations at the end of any period t must satisfy:

$$E_t^P \pi_{t+j} = E_t^P \pi_{t+j+1} - \kappa_{t|t} E_t^P x_{t+j} + \rho^j \xi_{t|t}^p$$

From the "lean against the wind" targeting rule, their unemployment gap expectations are:

$$E_t^P x_{t+j} = \frac{\kappa_{t|t}}{\lambda_t^P + \kappa_{t|t}^2} (E_t^P \pi_{t+j+1} + \rho^j \xi_{t|t}^p - \pi_t^{*P}) \tag{27}$$

Solving forward, we get analytical expressions describing their inflation and unemployment gap expectations for all future horizons in terms of their beliefs $\kappa_{t|t}, \xi_{t|t}^p, \pi_t^{*P}, \lambda_t^P$ formed conditional on $\mathcal{H}^t$:

$$E_t^P \pi_{t+j} = \underbrace{\pi_t^{*P}}_{\text{perceived target}} + \underbrace{\frac{\lambda_t^P}{\lambda_t^P(1-\rho) + \kappa_{t|t}^2} \rho^j \xi_{t|t}^p}_{\text{cost-push inflation}} \tag{28}$$

$$E_t^P x_{t+j} = \frac{\kappa_{t|t}}{\lambda_t^P(1-\rho) + \kappa_{t|t}^2} \rho^j \xi_{t|t}^p \tag{29}$$

Their expectations reflect simple economic intuition. According to Equation 28, from the private-agents' perspective, the path of inflation is ultimately determined by two forces: (i) π_t^{*P}, rep-

[13] The expression $\frac{\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}$ is bounded above by $\frac{1}{2\sqrt{\lambda_t}}$. So even though it also depends on κ_t, its limiting behavior is determined by λ_t.

resenting their beliefs about the central bank's inflation target, defines the level that the agents expect inflation to converge to in the long run. Inflation expectations at all horizons move one-for-one with π_t^{*P}. Hence this belief is a crucial determinant of the low-frequency (slow-moving) dynamics of inflation; and (ii) persistent supply shocks $\xi_{t|t}^p$, which create involuntary inflation above and beyond this voluntary level in the short-run: the extent of their *pass-through* to short-run inflation expectations is *decreasing* in the perceived slope of the Phillips curve $\kappa_{t|t}$ but *increasing* in λ_t^P, the perceived weight that the central bank puts on unemployment-gap stabilization. Thus, when agents perceive the central bank to prioritize real-side stabilization (λ_t^P is large), their short-run inflation expectations respond to persistent supply shocks strongly and causes them to depart further away from the long-run level, π_t^{*P}.

Perceived policy rule: The interest-rate i_{t+1}^* that implements the optimal unemployment gap plan is given by the IS-curve:

$$i_{t+1}^* = \tau^{-1}(x_{t+1|t} - E_t^{P-} x_{t+2}) + E_t^{P-}\pi_{t+2} + r_{t+1|t}^n$$

The agents' expectations $E_t^{P-}\pi_{t+j}$ and $E_t^{P-}x_{t+j}$, given their ex-ante beliefs about policymakers' preferences are given by:

$$E_t^{P-}\pi_{t+j} = \pi_{t-1}^{*P} + \frac{\lambda_{t-1}^P}{\lambda_{t-1}^P(1-\rho) + \kappa_{t|t}^2}\rho^j \xi_{t|t}^p$$

$$E_t^{P-}x_{t+j} = \frac{\kappa_{t|t}}{\lambda_{t-1}^P(1-\rho) + \kappa_{t|t}^2}\rho^j \xi_{t|t}^p$$

We obtain the following expression mapping the policymakers' period-t preferences to the optimal nominal interest rate:

$$i_{t+1}^*(\pi_t^*, \lambda_t) = \underbrace{-\tau^{-1}E_t^{P-}x_{t+2} + r_{t|t}^n + E_t^{P-}\pi_{t+2}}_{\text{ex-ante known}} + \underbrace{\frac{\tau^{-1}\kappa_{t|t}}{\lambda_t + \kappa_{t|t}^2}(E_t^{P-}\pi_{t+2} + \rho\xi_{t|t}^p - \pi_t^*)}_{\text{contains information about Fed objectives}}$$

When the central bank sets interest rates, it reveals information regarding its policy preferences. This is the only relevant information that interest rates reveal, since all the other variables in Equation 4.2 are known ex-ante. Only the last term depends on the policymakers' true policy preferences π_t^* and λ_t and therefore contains new information. The coefficient $\frac{\tau^{-1}\kappa_{t|t}}{\lambda_t+\kappa_{t|t}^2}$ partly controls the strength of the signaling channel. Observe that as λ_t increases, this term shrinks and the signal becomes weak.

In the limiting case of $\lambda_t \to \infty$, interest rates carry no independent information: systematic policy is entirely determined by the private agents' ex-ante expectations and the perceived natural rate of interest. As a result, the "learning channel" of policy completely dies out. Consequently, when agents perceive the central bank to only care about unemployment gap stabilization, they expect the unemployment gap to be zero at all future horizons and the central bank is seen to implement a Wicksellian policy: with the the ex-ante real interest rate $i^*_{t+1} - E^{P-}_t \pi_{t+2}$ tracking the perceived natural rate of interest $r^n_{t|t}$ closely. In this case, policymakers are expected to tolerate arbitrary deviations in inflation: there is no inflation-target and policy contains no information about the long-run level of inflation, leading to self-fulfilling dynamics. Note that policy response to $E^{P-}_t x_{t+2}$ and $r^n_{t|t}$ does not depend on λ_t - since they don't pose a trade-off.

4.3 Monetary policy surprises

The agents assume that the central bank sets interest rates optimally every period up to some idiosyncratic policy error of constant variance so that[14]:

$$i_{t+1} = i^*_{t+1} + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

Whenever the nominal interest rate is different from what the private agents expect from systematic policy, given their beliefs about prevailing macroeconomic conditions, it constitutes a monetary policy surprise:

$$\underbrace{i_{t+1} - E^{P-}_t i_{t+1}}_{\text{monetary policy surprise}} = \underbrace{(i^*_{t+1} - E^{P-}_t i^*_{t+1})}_{\text{shift in systematic policy}} \quad + \quad \underbrace{\epsilon^i_t}_{\text{idiosyncratic policy error}}$$

Agents interpret policy surprises as stemming from two sources: shifts in policymakers' preferences, which they must learn about, or a idiosyncratic policy errors carrying no information about future policy behavior. But they cannot tell whether a given surprise reflects a goal-oriented shift in policy or an unintended error.

[14]The source of this error could be motivated as coming from cognitive noise in decision-making. It could also be thought of as a "control error". The main idea being that the private agents allow for idiosyncratic deviations from optimal policy arising from factors unrelated to changes in the policymakers' stabilization preferences.

4.4 The nonlinear filter

The observed nominal interest rate functions as a noisy signal about the central bank's preferences $[\pi^*_t, \lambda_t]$, which describe the relevant state that the agents learn about:

$$i_{t+1} = i^*_{t+1}(\pi^*_t, \lambda_t) + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

Linearizing around agents' prior estimates $\pi^P_{t-1}, \lambda^P_{t-1}$ assuming $\pi^*_t \approx \pi^P_{t-1}, \lambda_t \approx \lambda^P_{t-1}$[15]

$$i^*_{t+1}(\pi^*_t, \lambda_t) \approx i^*_{t+1}(\pi^{*P}_{t-1}, \lambda^P_{t-1}) + \frac{\partial i^*_{t+1}}{\partial \pi^*_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\pi^*_t - \pi^{*P}_{t-1}) + \frac{\partial i^*_{t+1}}{\partial \lambda_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\lambda_t - \lambda^P_{t-1})$$

The monetary policy surprise, $\Delta_t = i_{t+1} - i^*_{t+1}(\pi^{*P}_{t-1}, \lambda^P_{t-1})$ is:

$$\Delta_t = \frac{\partial i^*_{t+1}}{\partial \pi^*_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\pi^*_t - \pi^{*P}_{t-1}) + \frac{\partial i^*_{t+1}}{\partial \lambda_t}|_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}(\lambda_t - \lambda^P_{t-1}) + \epsilon^i_t, \quad \epsilon^i_t \sim N(0, \sigma^2_i)$$

The approximation above (called the Extended Kalman Filter) yields an expression for i^*_{t+1} that is linear in the latent states π^*_t and λ_t much like the simple Kalman filter. The key difference is that the coefficients mapping the observable i^*_{t+1} to the latent states are not constant, but state-dependent. They are determined by the partial derivatives of the interest rate rule with respect to π^*_t and λ_t evaluated at the agents' prior estimates π^{*P}_{t-1} and λ^P_{t-1}. These determine how "informative" interest rates are perceived to be regarding the policymakers' stabilization preferences.

The agents perceive $\log \pi^*_t$ and $\log \lambda_t$ to evolve exogenously as independent random walks, so that[16]:

$$\begin{pmatrix} \log\ \pi^*_t \\ \log\ \lambda_t \end{pmatrix} = \begin{pmatrix} \log\ \pi^*_{t-1} \\ \log\ \lambda_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon^{\pi^*}_t \\ \epsilon^{\lambda}_t \end{pmatrix}, \qquad \begin{pmatrix} \epsilon^{\pi^*}_t \\ \epsilon^{\lambda}_t \end{pmatrix} \sim N(\mathbf{0}, \begin{pmatrix} \sigma^2_{\pi^*} & 0 \\ 0 & \sigma^2_{\lambda} \end{pmatrix})$$

Every period, agents update their beliefs π^{*P}_t and λ^P_t based on their best-estimates as:

$$\pi^{*P}_t = \pi^*_{t|t}, \quad \lambda^P_t = \lambda_{t|t}$$

[15]This assumption is important for the filtering problem to be well-specified. Since interest-rate dynamics would be state-dependent due to the nonlinearity of i^*_{t+1}, large discrepancies can cause the filter to diverge. The assumption implies that under their own beliefs, the degree of information asymmetry remains small and they track policymakers' goals efficiently at all times. In Appendix H, I report evidence in support of this assumption based on the Fed's Greenbook forecasts.

[16]This reparameterization ensures that π^*_t and λ_t are non-negative.

4.5 What anchors long-run beliefs?

Anchoring reflects the entrenchment of expectations, whereby beliefs about the long-run future remain resilient to surprises. The following quote from Bernanke (2007) summarizes what anchoring means in the context of monetary policy:

> "...I use the term "anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored..."

In the model, private agents' long-run inflation expectations depend entirely on their beliefs regarding the central bank's inflation target, which they update based on observed policy behavior. Inflation expectations would be "well-anchored" if the *pass-through* from policy surprises to their perceived target is small. How agents update their beliefs regarding the target depends on how much information interest rates are perceived to carry.

4.5.1 The signaling effect of policy: An information-theoretic perspective

From the perspective of the agents, the interest rate i_{t+1} is an observation carrying information about time-varying parameters π_t^* and λ_t. The amount of information that can be extracted from a signal regarding a parameter is given by its Fisher information, denoted as $\mathcal{I}_t$:

$$\mathcal{I}_t = \frac{1}{\sigma_i^2}\mathbf{H}^\top \mathbf{H}_t, \quad \mathbf{H}_t = \left[\frac{\partial i^*_{t+1}(\pi_t^*, \lambda_t)}{\partial \pi_t^*}, \frac{\partial i^*_{t+1}(\pi_t^*, \lambda_t)}{\partial \lambda_t}\right]_{\pi^{*P}_{t-1}, \lambda^P_{t-1}}$$

$\mathbf{H}_t$ is the Jacobian of i^*_{t+1}, evaluated at their ex-ante beliefs π^{*P}_{t-1} and λ^P_{t-1}. $\mathcal{I}_t$ is decreasing in σ_i^2, the variance of the idiosyncratic noise term. The larger the amount of idiosyncratic noise, the less informative the signal is regarding systematic policy behavior. Interest rates only carry information about changes in the underlying policy preferences to the extent that they are sensitive to them. The Jacobian $\mathbf{H}_t$ denotes the perceived sensitivity of interest rates to changes in π_t^* and λ_t. Since σ_i^2 is constant, the Jacobian will govern how relevant, or informative an observed policy surprise is, regarding these changes. Consider the partial derivative with respect to π_t^*:

$$\frac{\partial i^*_{t+1}}{\partial \pi_t^*}\Big|_{(\pi^{*P}_{t-1}\lambda^P_{t-1})} = -\frac{\tau^{-1}\kappa_{t|t}}{\lambda^P_{t-1} + \kappa^2_{t|t}} \tag{30}$$

Equation 30 implies that interest rates and the inflation target have an inverse relationship. Thus when interest rates undershoot agents' ex-ante expectations, their perceived target rises.

$$\frac{\partial i^*_{t+1}}{\partial \lambda_t}|_{(\pi^{*P}_{t-1},t\lambda^P_{t-1})} = -\frac{\tau^{-1}\kappa_{t|t}}{\lambda^P_{t-1} + \kappa^2_{t|t}} \underbrace{(E^{P-}_t \pi_{t+2} + \rho\xi^p_{t|t} - \pi^{*P}_{t-1})}_{\chi_t} \tag{31}$$

The factor in parenthesis denotes the perceived nominal distortion faced by the policymakers as a result of agents' inflation expectations and persistent supply shocks. From the policy rule, we can see that λ_t only affect interest rates when such distortions cause a policy trade-off. The larger they are in magnitude, the more information interest rates reveal about how policymakers weigh their dual mandate.

Equation 31 shows that the degree to which interest rates respond to the wedge χ_t is decreasing in λ_t. When interest rates respond to offset supply shocks more strongly than agents anticipate, they lower their beliefs about λ_t.

The magnitude of information: The coefficient $-\tau^{-1}\kappa_{t|t}/(\lambda^P_{t-1} + \kappa^2_{t|t})$ is common to both components of $\mathbf{H}_t$ and effectively acts as a scaling factor, determining the total amount of information that the interest rate signal carries. If agents believe this term to be small, they perceive a poor *signal-to-noise ratio* in the interest rate signal and the "learning channel" of monetary policy slows down. The expression implies that the degree to which agents stick to their prior beliefs about the Fed's policy preferences is increasing in λ^P_{t-1}. The more weight the agents perceive policy to attach to real-side stabilization, the less volatile their beliefs about the inflation target are. It determines how sensitive the agents' long-run inflation expectations are to monetary policy surprises. Henceforth, I denote this coefficient as ζ_t.

The direction of $\mathbf{H}_t$: The orientation of $\mathbf{H}_t$ determines the direction in the $\pi^* - \lambda$ space that the observation i_{t+1} "points" towards. The orientation of $\mathbf{H}_t$ is determined by the ratio of the partials, given by the distortion wedge χ_t. It can be shown that:

$$\chi_t = \frac{(\lambda^P_{t-1} + \kappa^2_{t|t})\rho\xi^p_{t|t}}{\lambda^P_{t-1}(1-\rho) + \kappa^2_{t|t}} \tag{32}$$

χ_t determines how a policy surprise loads on to beliefs about λ_t and π^*_t. When χ_t is large, agents perceive the surprise to contain more information about λ_t than π^*_t. Clearly, when $\chi_t = 0$, the policy surprise is seen to carry no information about λ_t since there is no policy trade-off. In this case, only π^{*P}_t gets updated.

5 Estimation

To characterize the path of beliefs and expectations, we must pin down the vector of parameters:

$$\Phi = [\tau, g_u, \rho, \sigma^2_{\xi,p}/\sigma^2_{\xi,tr}, \sigma^2_i, \sigma^2_{\pi^*}, \sigma^2_\lambda]$$

These parameters govern how private agents update beliefs conditional on observations. Given a vector of parameters Φ and a history of realizations $\mathcal{H}^t$, the path of private-sector beliefs regarding the state of the economy $\{\kappa_t, \xi^p_t, u^n_t, r^n_t\}$ as well as the central bank's policy preferences $\{\pi^*_t, \lambda_t\}$ are determined under the learning algorithm described earlier.

5.1 Data

The learning algorithm is implemented with monthly data on Headline CPI Inflation (CPIAUCSL), Unemployment rate (UNRATE) and the Effective Fed Funds rate (FEDFUNDS) from 1958:01-2024:12 (available from FRED St Louis) as observations. For the zero-lower bound period, the Wu-Xia shadow interest rate (Wu and Xia (2016)) is used instead of the Fed funds rate. To operationalize the filtering algorithm, one must specify the initial beliefs regarding the state variables $\{\kappa_t, \xi^p_t, u^n_t, r^n_t, \pi^{*P}_t, \lambda^P_t\}$. I specify plausible priors for these initial beliefs and allow them to adjust in response to the first 48 sample periods of data (corresponding to 4 years covering 1958:01-1961:12). These samples are then discarded and the filter is run forward in time at the end of the initialization period i.e beginning at 1962:01.

To discipline the estimates, I use survey data on long-run inflation expectations from the Survey of Professional Forecasters (SPF)—specifically, the cross-sectional mean of the 10-year-ahead CPI inflation forecasts—as a proxy for private agents' beliefs about the Federal Reserve's inflation target.[17] The SPF series is available at a quarterly frequency over the period 1991–2024 and is linearly interpolated to derive a monthly measure for ease of estimation. The implicit assumption here is that professional forecasters' long-run inflation expectations are broadly representative of those held by firms and households. Given the low volatility and slow-moving dynamics of long-run inflation expectations during this period, this assumption is reasonable.[18]

[17] Short-run inflation forecasts depend upon a combination of π^{*P}_t, λ^P_t and $\xi^p_{t|t}$, which makes identification much more difficult. Long-run inflation expectations have the advantage that they isolate shifts in π^{*P}_t alone.

[18] For a relatively short sample covering the recent post-pandemic episode, data from Survey of Firms' Inflation Expectations (SoFIE) from the Cleveland Fed is available, which explicitly includes firm managers' perceptions about the Fed's target. This series is in extremely close agreement with the SPF series used.

Although alternative measures covering a longer sample period exist such as the Michigan Survey of Consumers and Cleveland Fed's 10-year ahead inflation expectations series, they are not used in the baseline estimation. However, they are employed in an out-of-sample validation test (see Appendix A) to assess the robustness of the results. Not explicitly including long-run inflation expectations data for the Great Inflation period allows us to test if the structural dynamics captured by the model can provide a consistent explanation for the expectations-formation process during this period without explicitly being fit to the data.

5.2 Estimation results

The goal of estimation is to pin down the vector Φ that governs how the agents update their beliefs conditional on macroeconomic observations. I estimate Φ by minimizing the sum of the mean-squared discrepancies between the agents' long-run inflation expectations, given by π_t^{*P} and observed long-run inflation expectations from the SPF, and the discrepancies in their one-period ahead inflation forecasts from actual inflation. This allows us to discipline the model so that it replicates the dynamics observed in survey measures of long-run forecasts as well as generating expectations that remain close to the relevant target. The minimum-distance estimate Φ^* is the solution of the resulting minimization problem:

$$\Phi^* = \underset{\Phi}{\operatorname{argmin}} \left[\frac{1}{T_1} \sum_{t=1991:01}^{2024:12} \left([\pi_t^{*P} | \mathcal{H}^t, \Phi] - \pi_t^{10y,SPF} \right)^2 + \frac{1}{T_2} \sum_{t=1962:01}^{2024:12} \left([E_t \pi_{t+1} | \mathcal{H}^t, \Phi] - \pi_{t+1} \right)^2 \right]$$

Here, T_1 and T_2 denote the respective sample lengths. I discard values of Φ that generate explosive dynamics for any of the beliefs. This problem arises because beliefs are a non-differentiable function of Φ: even small perturbations in the parameters Φ can induce large changes in the updating process across all variables, setting off some beliefs on divergent paths, since all updates are made conditional on others.[19]

The point-estimate of Φ^* along with 95% credibility intervals generated using Sequential Monte Carlo (SMC) sampling are shown in Table 2. The point-estimate implies that the variance of innovations in the policy preference parameters is very small compared with the idiosyncratic

[19]This also makes the use of gradient-based optimization algorithms infeasible. I use the Nelder-Mead direct search method that does not require the computation of gradients or derivatives of the objective function. The search runs for about 20,000 iterations and completes in about 30 minutes on a M1 Macbook Air with 8 GB of unified memory.

policy errors. The innovations to persistent cost-push shocks are also approximately 10^3 times smaller than i.i.d. innovations.

Table 1: Estimated parameters of the baseline model

Parameter	Description	Estimate (95% CI)
τ	Ratio of IES to Okun's law coefficient	$0.647\ (0.430,\ 1.184)$
g_u	Gain for updating u_t^n	$0.329\ (0.209,\ 0.351)$
ρ	AR(1) persistence of supply shocks ξ_t^p	$0.984\ (0.979,\ 0.99)$
$\frac{\sigma^2_{\varepsilon^{tr}}}{\sigma^2_{\varepsilon^p}}$	Ratio of the variance of innovations to the transitory component of cost-push shocks to that of the persistent component	$755.901\ (496.125,\ 2176.436)$
σ_i^2	Variance of idiosyncratic monetary policy errors	$0.488\ (0.389,\ 1.334)$
$\sigma^2_{\pi^*}$	Variance of innovations to (log)inflation target	$6.696\ (5.186,\ 7.772) \times 10^{-4}$
σ^2_{λ}	Variance of innovations to (log)weight on unemployment-stabilization	$6.802\ (3.611,\ 9.173) \times 10^{-5}$

Note: Confidence intervals are constructed using Sequential Monte Carlo sampling. The likelihood function is not differentiable in the parameters Φ, so asymptotic standard errors cannot be used.

Appendix B compares the agents' ex-ante forecasts of inflation, unemployment rate and the nominal interest rate informed by their model with the actual realizations to assess the fit. The agents' perceived model is able to track the target variables closely and leads to surprises of small magnitude.

6 Evolution of the agents' beliefs

This section discusses time-paths of private-sector agents' beliefs regarding the structural variables describing the economy as implied by the point estimates given in Table 2. The section also provides context on how beliefs evolved and their bearing on the macroeconomic outcomes in the post-war period.

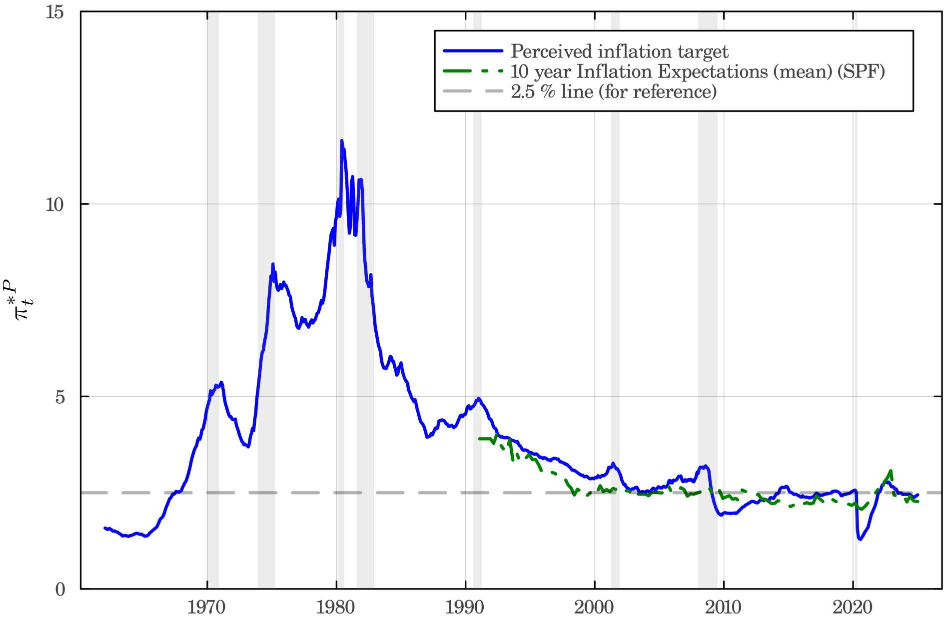


Figure 1: **Perceptions about the Fed's inflation target**

Note: Private agents' beliefs regarding the Fed's inflation target as implied by the model (π_t^{*P}) *(solid)* vs SPF 10-year inflation expectations *(dashed)* which serves as the empirical proxy. During the Great inflation, private-sector beliefs about the inflation target exhibit sharp upward revisions. This is not the case during the recent surge. The shaded regions indicate NBER recessions. The SPF 10-year ahead inflation expectations series begins in 1991:Q1 and is obtained from the Philadelphia Fed website.

6.1 The Fed's inflation target

Figure 1 illustrates the model-implied path of the agents' perceptions regarding the Fed's inflation target. The beliefs exhibit a significant degree of volatility prior to the mid-1980s, with sizable high-frequency movements.[20]According to the model, the private agents perceive the Fed's inflation target to be close to 2% in the early 1960s. In the latter half of the decade, the perceived target begins to rise, sitting above 4% by 1970.[21]

From the eyes of the private sector, the 70s would be a period of great volatility in the Fed's inflation target. Their beliefs are sharply revised upwards on two occasions, each coinciding with an NBER recession—one during the beginning of the decade and the other around the time of the OPEC Embargo and the oil crisis of 1974. From the lens of the model, the private agents saw that the Fed had an *incentive* to inflate the economy, over and beyond any supply disturbances

[20]This is in contrast with papers such as Bhattarai, Lee and Park (2016) that rely on identifying the inflation target from low-frequency movements in inflation and identify smooth variation.

[21]Hence I find that the anchor had begun to loosen in the late 1960s, before the appointment of Arthur Burns (in 1970), similar to Reis (2021). In December 1965 a policy-tightening provoked an unprecedented backlash against Fed Chair Martin, marking what many observers regard as the beginning of a prolonged era of accommodative monetary policy.

creating policy trade-offs. From their point of view, the Fed displayed a tendency to deliberately inflate during recessions, operating with a higher inflation target, followed by dis-inflationary pursuits soon after. The Volcker-era, beginning with the chairmanship of Paul Volcker in 1979 is often cited as watershed moment in the Fed's eventual "conquest" of inflation. In October 1979, the Fed announced "new operating procedures" aimed at reining in inflation. But these do not immediately effect a disinflation (as also noted by Goodfriend and King (2005)). A recession follows and the perceived target shoots up yet again. It is not until 1982 that we see the perverse dynamics reverse course. The following years see a sharp decline in the perceived inflation target which coincides with a deep recession. By the mid 1980s, agents perceive the Fed's inflation target to be below 5% after nearly a decade. Thus the U.S. economy enters the "Great Moderation". Beliefs regarding the Fed's target are on a steady downward trajectory in the two decades that follow, under the chairmanship of Alan Greenspan (1987-2006). Only during the Great Financial crisis and the recent COVID-pandemic do we see the agents' perceptions exhibiting some modest revisions (interestingly, breaking away from the SPF series).

The model implies a modest uptick in the agents' perceptions regarding the Fed's inflation target during the post-pandemic episode, also observed in the SPF data, showing that there was some endogenous feedback from monetary policy in the post-pandemic period to private agents' long-run inflation expectations. Yet, beliefs about the target only rise to about 3%. This is remarkable considering that CPI inflation itself reached a peak of 9.1% in June 2022. The same learning mechanism that caused agents to anticipate inflation to remain put at levels as high as 11% during the 80s, told them that inflation was on course to settling around 3% - the inflation crisis left their long-run beliefs more or less unchallenged.

While the model implies that the conduct of policy created the perception among private agents that the inflation target had risen substantially during the 1970s, the model does not take a stand on what would cause the policymakers to acquire a taste for such exorbitant levels of inflation. Narrative evidence offered by Romer and Romer (2002), DeLong (1996) provide some valuable clues: inflation was deliberately pursued under the false notion that unemployment could be driven lower by inflationary surprises. Bianchi and Ilut (2017) offer a view that inflation was fiscally-led during this period. This view would be consistent with policymakers pursuing higher inflation in order to accommodate the government's fiscal expansion, effectively helping to "inflate the debt away". This would suggest that the rise in the inflation target was politically motivated. The focus here is not on understanding why the policymakers operated in this way but rather, how policy behavior shaped private agents' expectations.

Figure 2 plots the agents' short-run inflation expectations (one-year ahead, as implied by the model) along with their long-run inflation expectations, given by their perceived inflation target.

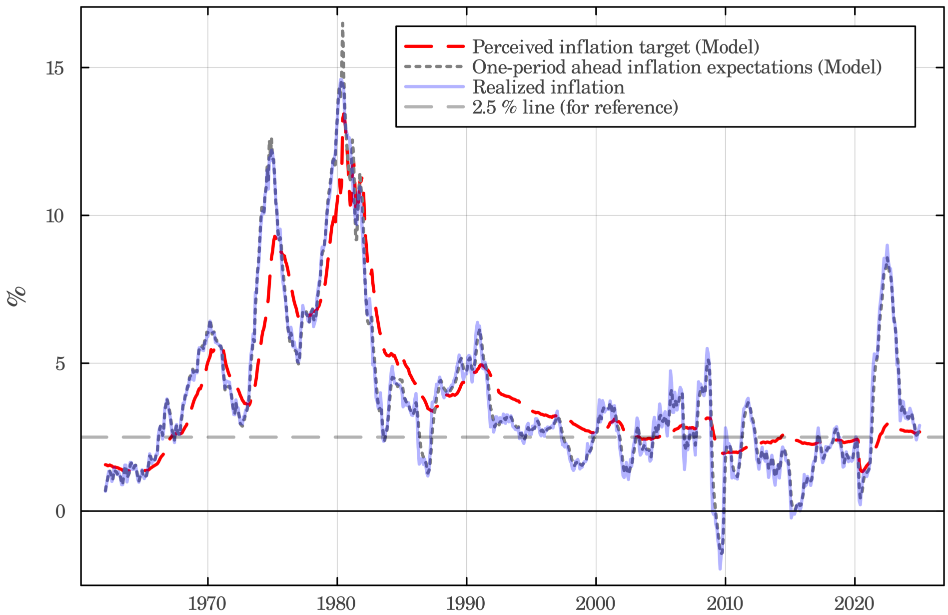


Figure 2: **Weakening link between short-run and long-run inflation expectations**

Note: The figure plots agents' model-implied one-year ahead inflation expectations (dotted line) along with the long-run inflation expectations (dashed line). Realized inflation is also plotted for reference (solid line, almost coincident with short-run expectations)

Owing to deflationary supply shocks corresponding to the oil glut in the mid-1980s, even as the agents expected inflation to be below 2.5% in the short-run, for the first time in nearly two decades, they still believed the Fed's inflation target to be near 4%. Hence, for a time, agents viewed the benign inflation as a temporary reprieve driven by the macroeconomic environment soon to be undone by policy. Over the recent period however, while short-run inflation expectations continued to be volatile (closely matching the dynamics of actual inflation), agents' beliefs about the target have held relatively stable, close to 2.5%. This shift has much to do with their perception regarding the Fed's emphasis on minimizing real-side volatility, as we will see in the next subsection.

6.2 The Fed's weight on unemployment gap stabilization

Figure 3 plots the evolution of the private sector's estimates of λ_t: the relative weight the Fed puts on closing the unemployment-gap. The estimates suggest that, during the early 1960s, the private sector attributed relatively little concern from the Fed towards closing the gap.

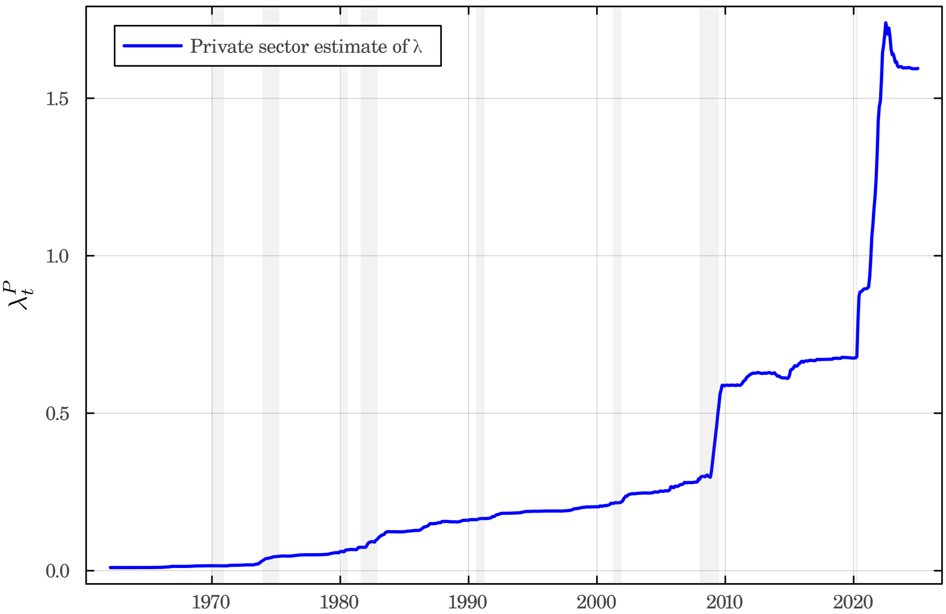


Figure 3: **Perception of the Fed's weight on unemployment-gap stabilization**

Note: Evolution of the Private sector estimate of λ_t. The private sector perceives the Fed to put more emphasis on stabilizing the unemployment-gap. The shaded regions indicate NBER recessions

After a slow and steady rise following the Volcker disinflation, perceptions regarding λ were sharply revised upwards following the Global Financial Crisis and again in the aftermath of the COVID-19 pandemic: the emphasis of policy has shifted. As described in subsection 4.5, this development has had important implications for the anchoring of long-run inflation expectations. A large λ implies that the Fed effectively "ties its hands"; unemployment gaps are perceived as costly and hence policy emphasizes keeping the gap closed - constraining policymakers from responding to exogenous shifts in their implicit inflation target (owing to say, political pressure or short-term goals). A large λ_t may also be interpreted as the policymakers' increasing adherence to the natural rate of unemployment as the relevant policy target, since an increase in λ_t shifts policy focus away from the inflation target.

These findings are consistent with recent empirical literature using high-frequency identification utilizing changes in asset prices around Fed policy announcements to study how the private sector learns about the Fed's "policy rule" (e.g., Bauer and Swanson (2023), Bauer, Pflueger and Sunderam (2024), Pflueger (2025)). These studies find that monetary policy in the recent period, following 2000, has been perceived to be increasingly responsive to output. A rise in λ_t, and the greater predictability of unemployment-gaps also complements earlier findings that output-gap volatility in the U.S. has declined significantly since the Volcker disinflation (see Blanchard and Simon (2001); McConnell and Perez-Quiros (2000)), underscoring the role of systematic monetary policy in engineering this phenomenon.

6.3 The slope of the expectations-augmented Phillips Curve

The parameter κ, represents the sensitivity of aggregate inflation to contemporaneous unemployment gap. Hence, it serves as the connective tissue between the real and nominal side of the economy and is a key parameter that agents must learn. Figure 4 plots the evolution of the agents' beliefs regarding the slope of the expectational Phillips curve under their learning mechanism. Under the agents' prior, κ is time-invariant. However, their real-time estimates under learning exhibit significant time-variation over the sample period.

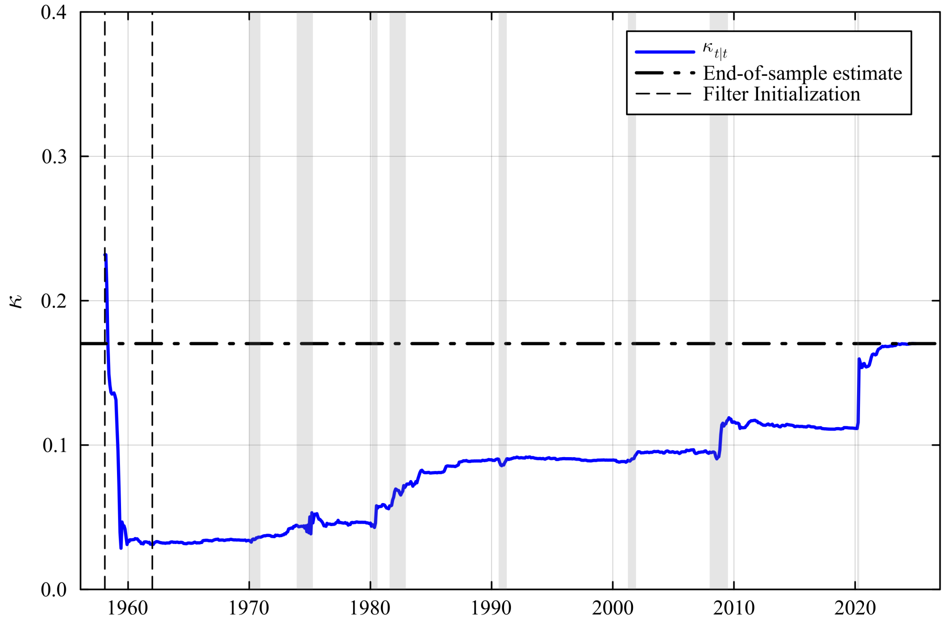


Figure 4: **Slope of the expectations-augmented Phillips curve**

Note: The figure plots the filtered estimates of the slope of the Phillips curve (*solid line*). From the private agents' perspective, the Phillips curve is steeper now than it was during the 70s and the 80s, meaning that inflation is *more* sensitive of real activity. Interestingly, though the agents learn under the assumption that the true slope of the structural Phillips curve is time-invariant, their beliefs are subject to constant revisions owing to the identification problems posed by the information structure of the economy. The shaded regions indicate NBER recessions

Since κ is not subject to innovations, the steady-state filter would be a decreasing-gain one. As new data is observed and beliefs are updated, each additional observation carries marginally less information and their beliefs should stabilize near a constant value. But it does not appear that any such convergence has attained within the sample period spanning about 60 years.

To understand why this is the case, let's revisit the observation equation through which agents update their beliefs regarding κ and ξ_t^p:

$$\pi_t - E_{t-1}^P \pi_{t+1} = -\kappa x_t + \xi_t^p + \xi_t^t$$

Note that current inflation, π_t delivers information about κ only through the independent variation in the unemployment gap x_t. It is not surprising then, that we see the bulk of the large updates in agents' estimates of κ around recessions (causing agents to perceive large demand shocks). During "normal" times, the unemployment gap doesn't offer much variation and it is evident that learning about κ would slow down. The signal informing the agents about κ is susceptible to getting drowned out owing to aggregate supply shocks. Notably, the unemployment gap x_t is of small magnitude most of the time (Figure 13 in Appendix E plots the perceived unemployment-gap as implied by the model over the post-war period). The mean absolute value of the gap over the sample period is 0.27; implying that even if the "true" slope were twice as large as the agents' end-of-sample-estimate, it would, on average create a discrepancy of only about $\approx 0.05\%$ in aggregate inflation. These challenges are outlined in Mavroeidis, Plagborg-Møller and Stock (2014) who conclude that aggregate time series data can offer only limited insight into the nature of the Phillips curve. Indeed, learning about the Phillips curve relationship has been among the most enduring and difficult problems in macroeconomics. The agents in the model are not immune to the fundamental econometric challenges that learning poses. Models assuming full-information on part of agents would obscure these effects.

The agents perceive the slope to be relatively "flat" during the 1970s. This belief would be reinforced by the experience of high inflation in the face of rising unemployment experienced during this period. The perception of a flat Phillips curve is one of the primary reasons cited by Primiceri (2006) and Sargent, Williams and Zha (2006) for the policymakers' aversion to act decisively against inflation during this period. On the other hand, in the post-pandemic period, the sudden decline in inflation from its highs with little rise in unemployment leads the agents to revise their beliefs about κ upwards.

However, estimates during the entire sample period remain within an empirically plausible range (between 0.01-0.20), implying that the agents do not perceive the slope to be excessively flat at any point. Recent papers argue that modeling nonlinearities in the Phillips curve could be important to understand the behavior of inflation during the post-pandemic period (see Harding, Lindé and Trabandt (2023) Benigno and Eggertsson (2024)). Consistent with these papers, I do find that the agents' expectations reflect a belief that the Phillips curve has steepened, following the pandemic.

6.4 Sensitivity of long-run inflation expectations

The coefficient, $\zeta_t = \frac{\tau^{-1}\kappa_{t|t}}{\lambda^P_{t-1}+\kappa^2_{t|t}}$ which depends both on the agents' perceived slope of the Phillips curve, $\kappa_{t|t}$ as well as their ex-ante perceptions about the weight that the Fed puts on unemployment gap stabilization, λ^P_{t-1} determines the degree to which interest rates are perceived to be informative about shifts in the inflation target. This coefficient also determines how sensitive agents' long-run inflation expectations would be to monetary policy surprises. Figure 5 plots the evolution of ζ_t over the postwar period.

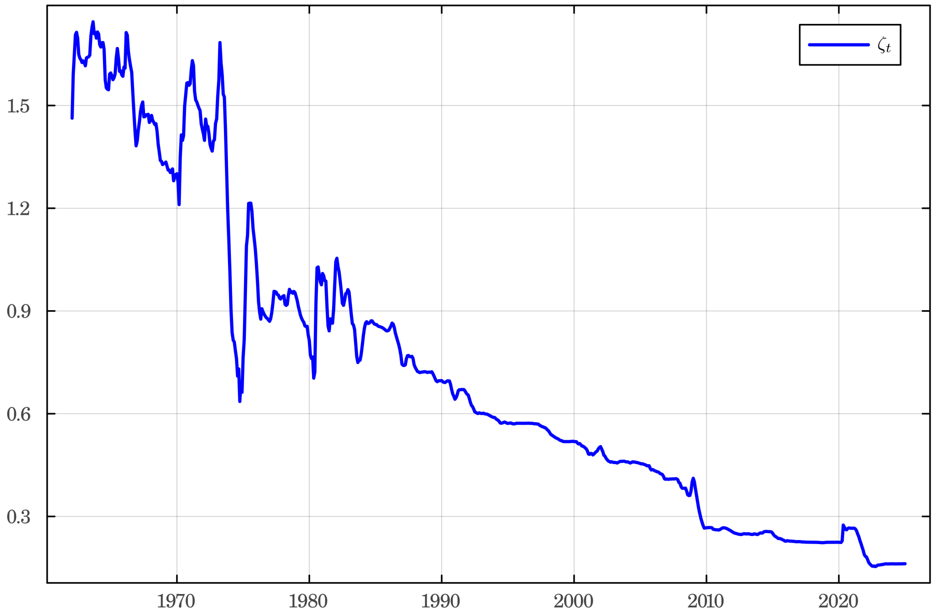


Figure 5: **Evolution of the anchoring coefficient** (ζ_t)

Note: The coefficient measuring the sensitivity of long-run inflation expectations to monetary policy surprises has fallen drastically, driven by the Fed's demonstrated emphasis on minimizing real volatility.

From the private agents' vantage point, the policy environment has changed drastically in the 40 years separating the Great Inflation and the post-pandemic surge. Through the lens of their model, the agents perceive interest rates to be approximately five times less sensitive to shifts in the inflation target than they were in the wake of the Great Inflation crisis. The model also implies that their long-run inflation expectations are more entrenched now than any other time in the post-war history of the United States. This development afforded the Fed enough room to to "look-through" supply shocks without triggering large revisions in long-run inflation expectations. While this lowers the risk of de-anchoring, it also hampers the Fed's ability to guide these beliefs back to target *if* they were to de-anchor. Thus, according to the model, the "final mile" of bringing inflation back to below 2% would take longer than if the agents held the same beliefs about policy as they did during the 1980s.

6.5 "Bad luck" vs "bad policy" revisited

The following section contributes to the debate regarding whether the economic outcomes of the post-Volcker period were a result of luck (favorable economic environment, benign shocks) or systematic policy (shifts in the Fed's implied policy objectives). I discuss how evolving perceptions regarding structural shocks and the Fed's systematic monetary policy contributed to shaping macroeconomic outcomes in the post-war period in the U.S.

6.5.1 The role of policy in shaping agents' expectations

Monetary policy surprises function as signals that shape private sector expectations. Figure 6 displays the model-implied surprises for the post-war U.S. sample.

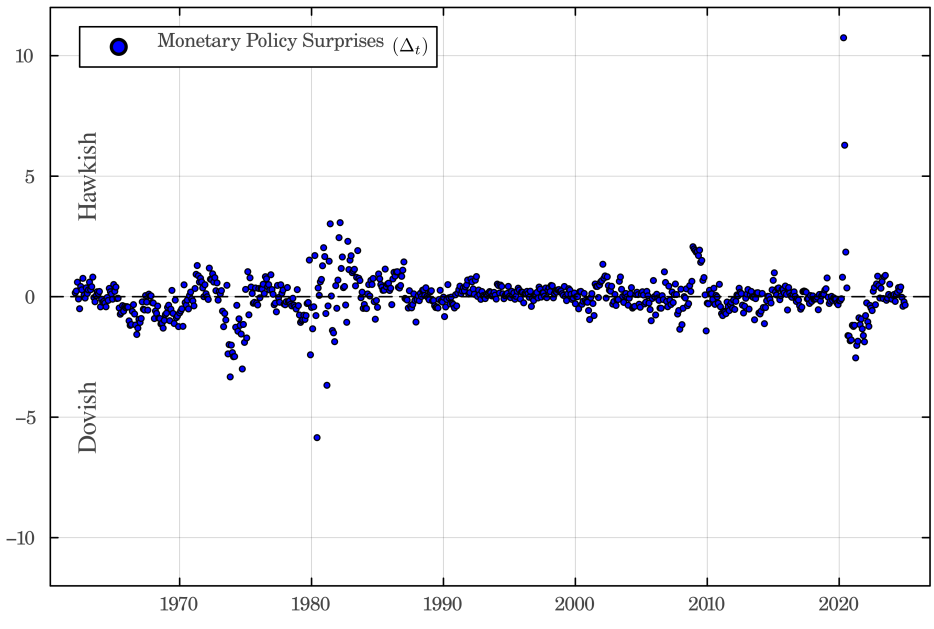


Figure 6: **Monetary policy surprises**

Note: The figure plots the policy surprises perceived by the agents as they interpret the Fed's policy decisions in real-time. Positive surprises represent hawkish stance.

In the late 1960s, a sequence of dovish surprises suggests to the agents that the Fed's implicit inflation target is rising. Given that long-run inflation expectations were highly sensitive to policy surprises during this period—as evidenced by the anchoring coefficient discussed previously—they displayed sharp upward revisions. This pattern re-emerged in the mid-1970s following the OPEC embargo and subsequent oil crisis; policy shocks during this era further reinforced the perception of a higher target.

It was not until the early 1980s that the agents see a reversal to a hawkish stance. Under Paul Volcker's chairmanship, persistent hawkish surprises served to signal a shift in the Fed's inflation preferences. Consequently, through the learning mechanism, both realized inflation and long-run expectations declined sharply. Notably, the model suggests the Volcker Fed maintained a hawkish posture in the eyes of agents even as the pressure from supply shocks abated through the mid-1980s.

Post-1990, Fed policy became largely predictable, with shocks remaining small in magnitude. However, the model yields a few striking results regarding recent crises: it implies that agents perceived the Fed as overly hawkish during the 2008 Great Financial Crisis. [22] Conversely, the post-pandemic period is characterized by large dovish surprises stemming from the delay in liftoff relative to rising inflation. While these recent shocks are comparable in magnitude to those of the mid-1970s, their impact on expectations differs fundamentally, as agents now interpret policy through the lens of evolved beliefs regarding the Fed's dual mandate.

6.5.2 Supply shocks: The perceived drivers of policy trade-offs

When the agents perceive persistent supply shocks, it causes their short-run inflation expectations to depart from their beliefs regarding the Fed's inflation target. Figure 7 plots the time-series of the supply shock ξ_t^p.

[22]Similarly, at the onset of the COVID-19 recession, sharp downward revisions in the perceived natural rate of interest (r_t^n) caused the Fed's stance to be interpreted as hawkish.

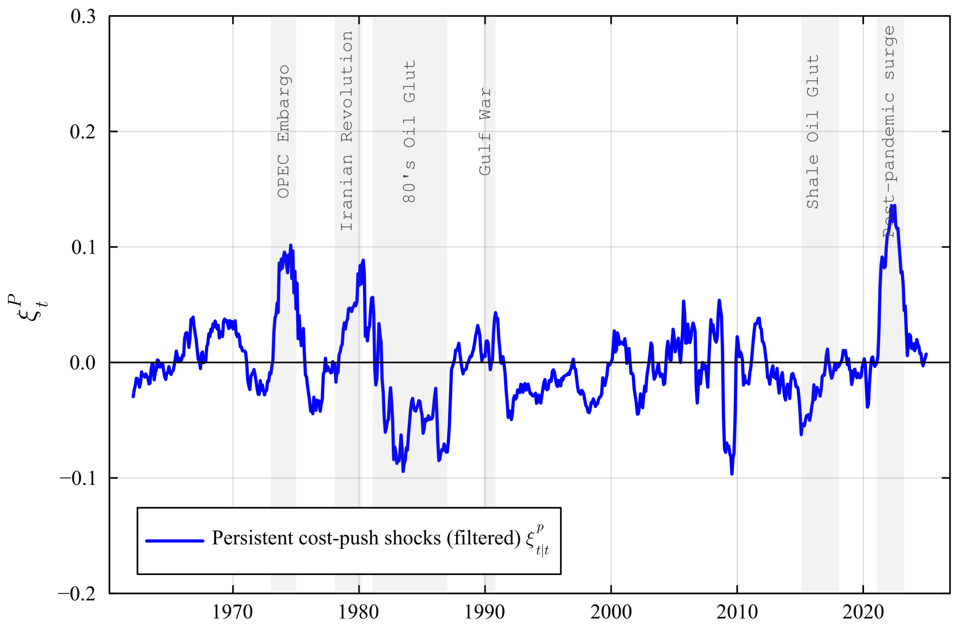


Figure 7: **Persistent supply shocks perceived by the agents**

Note: The figure plots the evolution of the agents' real-time beliefs regarding persistent aggregate supply shocks. The magnitude of persistent supply shocks perceived by the agents during the post-COVID period were similar to those during the Great Inflation period. The shaded regions indicate known episodes of significant supply pressures, providing some narrative context.

Reasoning about their macroeconomic environment, the agents perceive large persistent supply shocks, both during the Great Inflation of the 70s and also during the recent inflation surge. The perceived supply shocks line up well with documented episodes of oil price surges. There is ample evidence in the literature that fluctuations in relative prices of energy goods are significant drivers of inflation (recent papers include Afrouzi, Bhattarai and Wu (2024), Gagliardone and Gertler (2023)). Interestingly, the period of recovery from the Great Financial Crisis (GFC) also coincides with persistent cost-push shocks. This is consistent with the theory that oil price shocks in the aftermath of the crisis kept inflation expectations propped up, accounting for the "missing deflation" (Coibion and Gorodnichenko (2015)). According to the model, even as the agents' long-run inflation expectations remain stable during this period, their short-run expectations are quite responsive to inflationary supply shocks during this period.

To aid interpretation of these "persistent supply shocks", in Figure 8 I plot the series against food and energy price inflation derived as the difference between Headline CPI and Core CPI. The series are de-meaned and re-scaled for comparison. The two series display significant co-movement. Since Core CPI is not a series that the model has "seen" (only Headline CPI data is used during estimation), it is remarkable that such a link emerges.

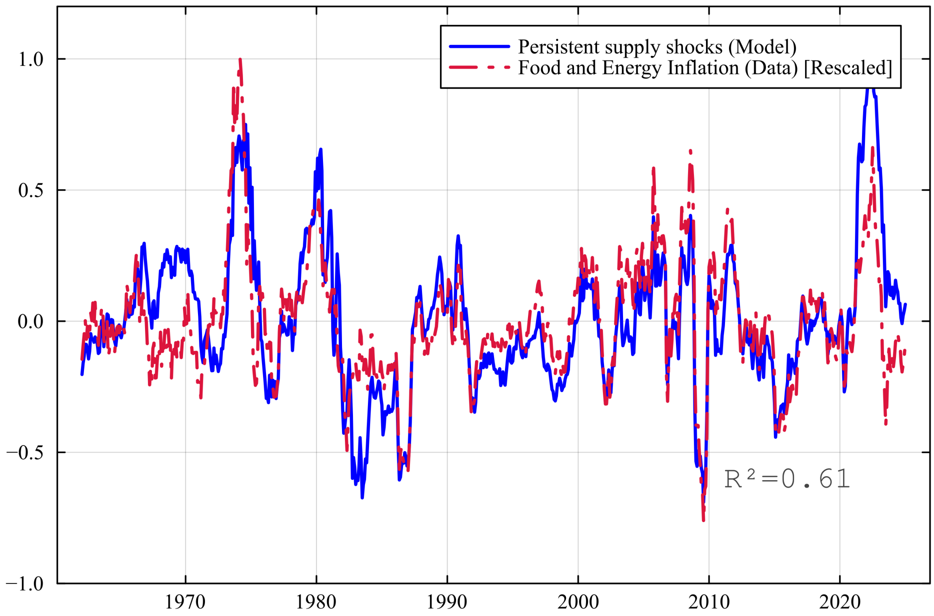


Figure 8: **Persistent supply shocks and relative prices**

Note: Persistent supply shocks as perceived by the agents vs inflation in food and energy prices (Untargeted). Measure of Food and Energy inflation is constructed as the difference between headline and core CPI inflation

The post-pandemic episode was characterized by structural shifts over and above oil price surges, such as sectoral shocks shifting consumption between goods and services (see Guerrieri et al. (2021)) structural changes in the labor market (Blanchard and Bernanke (2023), Ball et al. (2022)), increasing attention to inflation (Pfauti (2023)) to name a few. However, shocks to relative food and energy prices appear to explain a large part of the variation in these persistent supply shocks perceived by the agents. This is intuitive considering how *salient* these prices are (as also argued by Coibion and Gorodnichenko (2015)). It makes sense that firms and households' perceptions about inflation in the short run would respond to these prices given their pervasive impact on the rest of the economy (as argued in Afrouzi et al. (2024) using a model of input-output linkages). Hence, the model adds support to the literature emphasizing the importance of relative price disturbances in driving aggregate inflation dynamics.

7 Conclusions

The rational expectations revolution—spearheaded by the work of Lucas and Sargent—marked a profound shift in macroeconomic thought. New Keynesian macroeconomics incorporated this transformation by explicitly modeling how aggregate outcomes emerge from the optimizing behavior of individual agents. The model developed in this paper presents a natural extension of the standard New Keynesian framework to an environment characterized by imperfect knowledge where neither the true structure of the economy nor the objectives of the central bank policymakers are known to the private agents in the economy, and they use economic intuition to learn about the relevant variables in real-time. A fundamental break from the literature on learning is that here, agents learn while internalizing the forward-looking nature of economic dynamics and the endogeneity of macroeconomic outcomes with respect to their expectations. This allows for a much richer description of the feedback between macroeconomic dynamics and agents' learning.

A central implication of the model is that, because expectations are pivotal to shaping macroeconomic outcomes, the function of monetary policy extends beyond mechanical stabilization. Policy actions not only influence current economic conditions but also actively shape agents' expectations—a point emphasized by Kydland and Prescott (1977). The learning mechanism embedded in the model illustrates how the anchoring of expectations emerges endogenously from the systematic conduct of monetary policy. The evolution of postwar U.S. macroeconomic dynamics is thus closely linked to how Federal Reserve policymakers have signaled their policy objectives through the conduct of stabilization policy. The model provides a structural interpretation of how monetary policy contributes to the anchoring of expectations, highlighting its dual role as both a stabilizing instrument and a signal relaying information and affecting expectations.

A demonstrated emphasis by the Fed on minimizing policy-driven fluctuations in unemployment has contributed to the anchoring of long-run inflation expectations, *cushioning* people's long-run beliefs against policy misbehavior; while simultaneously amplifying the inflationary impact of persistent supply shocks. From the lens of the model, these perceptions afforded the Fed the ability to "look-through" supply shocks without de-anchoring long-run expectations. Beliefs shaped by the disciplined conduct of monetary policy over the forty years following the Volcker disinflation have contributed greatly to the "soft-landing" that we saw recently. While recent work (e.g., Coibion and Gorodnichenko (2025)) has warned of inflationary risks resulting from de-anchoring of expectations from inflationary pressures in the short-run, the model presented here paints a less grim picture while recognizing the risks that an indecisive policy response to supply shocks could pose in the future.

This work also highlights the explanatory power of the textbook New-Keynesian model when augmented with imperfect information and learning. The model reconciles the empirically observed persistence in macroeconomic aggregates, delivering an excellent fit to macroeconomic data without resorting to backward-looking mechanisms such as indexation to past prices, habit formation in consumption or interest rate smoothing. Finally, the model highlights the *state-dependent* nature of agents' learning: the rate at which information arrives is neither uniform over time nor identical across structural parameters. One key parameter for which this distinction is particularly relevant is the slope of the expectations-augmented Phillips curve, which remains poorly identified under learning. This finding suggests that the full-information assumption commonly imposed in macroeconomic models may be far less innocuous than often presumed.

References

Adam, Klaus and Mario Padula, "Inflation Dynamics and Subjective Expectations in the United States," *Economic Inquiry*, 2011, *49* (1), 13–25. 5

Afrouzi, Hassan, Saroj Bhattarai, and Edson Wu, "Relative-price changes as aggregate supply shocks revisited: Theory and evidence," *Journal of Monetary Economics*, 2024, *148*, 103650. Inflation in the COVID Era and Beyond. 36, 37

Ball, Laurence, Daniel Leigh, and Prakash Loungani, "Okun's Law: Fit at 50?," *Journal of Money, Credit and Banking*, 2017, *49* (7), 1413–1441. 13

Ball, Laurence M, Daniel Leigh, and Prachi Mishra, "Understanding U.S. Inflation During the COVID Era," Working Paper 30613, National Bureau of Economic Research October 2022. 37

Barro, Robert J. and David B. Gordon, "Rules, discretion and reputation in a model of monetary policy," *Journal of Monetary Economics*, 1983, *12* (1), 101–121. 6

Bauer, Michael D. and Eric T. Swanson, "A Reassessment of Monetary Policy Surprises and High-Frequency Identification," *NBER Macroeconomics Annual*, 2023, *37* (1), 87–155. 30

Bauer, Michael D, Carolin E Pflueger, and Adi Sunderam, "Perceptions About Monetary Policy*," *The Quarterly Journal of Economics*, 06 2024, *139* (4), 2227–2278. 30

Benigno, Pierpaolo and Gauti B. Eggertsson, "The Slanted-L Phillips Curve," *AEA Papers and Proceedings*, May 2024, *114*, 84–89. 32

Bernanke, Ben, "Inflation Expectations and Inflation Forecasting," July 10 2007. At the Monetary Economics Workshop of the National Bureau of Economic Research Summer Institute, Cambridge, Massachusetts. 22

Bernanke, Ben S. and Ilian Mihov, "Measuring Monetary Policy*," *The Quarterly Journal of Economics*, 08 1998, *113* (3), 869–902. 6

Bhattarai, Saroj, Jae Won Lee, and Woong Yong Park, "Policy Regimes, Policy Shifts, and U.S. Business Cycles," *The Review of Economics and Statistics*, 12 2016, *98* (5), 968–983. 27

Bianchi, Francesco and Cosmin Ilut, "Monetary/Fiscal policy mix and agents' beliefs," *Review of Economic Dynamics*, 2017, *26*, 113–139. 6, 28

Blanchard, Olivier and John Simon, "The Long and Large Decline in U.S. Output Volatility," *Brookings Papers on Economic Activity*, 2001, *2001* (1), 135–164. 30

Blanchard, Olivier J and Ben S Bernanke, "What Caused the US Pandemic-Era Inflation?," Working Paper 31417, National Bureau of Economic Research June 2023. 37

Bullard, James and Stefano Eusepi, "Did the great inflation occur despite policymaker commitment to a Taylor rule?," *Review of Economic Dynamics*, 2005, *8* (2), 324–359. Monetary Policy and Learning. 5

Carvalho, Carlos, Stefano Eusepi, Emanuel Moench, and Bruce Preston, "Anchored Inflation Expectations," *American Economic Journal: Macroeconomics*, January 2023, *15* (1), 1–47. 6

Christiano, Lawrence and Christopher Gust, "The expectations trap hypothesis," *Economic Perspectives*, 2000, *25* (Q II), 21–39. 6, 18

Clarida, Richard, Jordi Gali, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory*," *Quarterly Journal of Economics*, February 2000, *115* (1), 147–180. 6

Cogley, Timothy and Thomas J. Sargent, "Anticipated Utility and Rational Expectations as approximations of Bayesian decision making*," *International Economic Review*, February 2008, *49* (1), 185–221. 14

_ , Giorgio E. Primiceri, and Thomas J. Sargent, "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, January 2010, *2* (1). 5

Coibion, Olivier and Yuriy Gorodnichenko, "Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation," *American Economic Journal: Macroeconomics*, January 2015, *7* (1), 197–232. 36, 37

— **and** — , "Inflation, Expectations and Monetary Policy: What Have We Learned and to What End?," Working Paper 33858, National Bureau of Economic Research May 2025. 38

— **,** — **, and Rupal Kamdar**, "The Formation of Expectations, Inflation, and the Phillips Curve," *Journal of Economic Literature*, 2018, *56* (4), 1447–1491. 5

Debortoli, Davide and Aeimit Lakdawala, "How Credible Is the Federal Reserve? A Structural Estimation of Policy Re-Optimizations," *American Economic Journal.Macroeconomics*, 07 2016, *8* (3), 42–76. 6

Del-Negro, Marco and Stefano Eusepi, "Fitting observed inflation expectations," *Journal of Economic Dynamics and Control*, 2011, *35* (12), 2105–2131. Frontiers in Structural Macroeconomic Modeling. 5

DeLong, J. Bradford, "America's Only Peacetime Inflation: The 1970s," NBER Historical Working Papers 0084, National Bureau of Economic Research, Inc May 1996. 6, 28

Erceg, Christopher J. and Andrew T. Levin, "Imperfect credibility and inflation persistence," *Journal of Monetary Economics*, 2003, *50* (4), 915–944. 5, 47

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton University Press, 2001. 14

Gagliardone, Luca and Mark Gertler, "Oil Prices, Monetary Policy and Inflation Surges," Working Paper 31263, National Bureau of Economic Research May 2023. 36

Gati, Laura, "Monetary policy & anchored expectationsâ€"An endogenous gain learning model," *Journal of Monetary Economics*, None 2023, *140* (S), 37–47. 6

Goodfriend, Marvin and Robert G. King, "The incredible Volcker disinflation," *Journal of Monetary Economics*, July 2005, *52* (5), 981–1015. 28

Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Ivan Werning, "Monetary Policy in Times of Structural Reallocation," 2021. Proceedings of the Jackson Hole Symposium, 2021. 37

Harding, Martín, Jesper Lindé, and Mathias Trabandt, "Understanding post-COVID inflation dynamics," *Journal of Monetary Economics*, 2023, *140*, S101–S118. Inflation: Drivers and Dynamics 2022. 32

Ireland, Peter N., "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, 2007, *39* (8), 1851–1882. 5, 51

Kamber, Güneş, James Morley, and Benjamin Wong, "Intuitive and Reliable Estimates of the Output Gap from a Beveridge-Nelson Filter," *The Review of Economics and Statistics*, 07 2018, *100* (3), 550–566. 14

King, Robert G. and Yang K. Lu, "Evolving Reputation for Commitment: The Rise, Fall and Stabilization of US Inflation," NBER Working Papers 30763, National Bureau of Economic Research, Inc Dec 2022. 6

Kozicki, Sharon and P.A. Tinsley, "Permanent and transitory policy shocks in an empirical macro model with asymmetric information," *Journal of Economic Dynamics and Control*, 2005, *29* (11), 1985–2015. Expectations, learning, and monetary policy. 5

Kreps, David M., "Anticipated Utility and Dynamic Choice (1997)," in Donald P. Jacobs, Ehud Kalai, Morton I. Kamien, and Nancy L. Schwartz, eds., *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures, 1983–1997*, Econometric Society Monographs, Cambridge: Cambridge University Press, 1998, pp. 242–274. 14

Kydland, Finn E. and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, 1977, *85* (3), 473–491. 3, 38

Lakdawala, Aeimit, "Changes in Federal Reserve preferences," *Journal of Economic Dynamics and Control*, 2016, *70*, 124–143. 5

Lubik, Thomas A. and Frank Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, March 2004, *94* (1), 190–217. 6

Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H. Stock, "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, March 2014, *52* (1), 124–88. 32

McConnell, Margaret M. and Gabriel Perez-Quiros, "Output Fluctuations in the United States: What Has Changed Since the Early 1980's?," *The American Economic Review*, 2000, *90* (5), 1464–1476. 30

Melosi, Leonardo, "Signalling Effects of Monetary Policy," *The Review of Economic Studies*, 09 2016, *84* (2), 853–884. 5

Milani, Fabio, "Expectations, learning and macroeconomic persistence," *Journal of Monetary Economics*, 2007, *54* (7), 2065–2082. 47

—, "Chapter 18 - Expectational data in DSGE models," in Rüdiger Bachmann, Giorgio Topa, and Wilbert van der Klaauw, eds., *Handbook of Economic Expectations*, Academic Press, 2023, pp. 541–567. 5

Nakamura, Emi, Venance Riblier, and Jón Steinsson, "Beyond the Taylor Rule," August 2025. Manuscript, UC Berkeley. 3

Nimark, Kristoffer, "Dynamic pricing and imperfect common knowledge," *Journal of Monetary Economics*, 2008, *55* (2), 365–382. 10

Orphanides, Athanasios and John C. Williams, "The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations," *Journal of Economic Dynamics and Control*, 2005, *29* (11), 1927–1950. Expectations, learning, and monetary policy. 5

_ and _ , "Monetary Policy with Imperfect Knowledge," *Journal of the European Economic Association*, 2006, *4* (2/3), 366–375. 5

_ and Simon van Norden, "The Unreliability of Output-Gap Estimates in Real Time," *The Review of Economics and Statistics*, 2002, *84* (4), 569–583. 14

Pfauti, Oliver, "The Inflation Attention Threshold and Inflation Surges," Papers 2308.09480, arXiv.org Aug 2023. 37

Pflueger, Carolin, "Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds," *Journal of Financial Economics*, 2025, *167*, 104027. 30

Phillips, A. W., "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957," *Economica*, 1958, *25* (100), 283–299. 55

Primiceri, Giorgio E, "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy*," *Quarterly Journal of Economics*, August 2006, *121* (3), 867–901. 5, 32

Reis, Ricardo, "Losing the Inflation Anchor," *Brookings Papers on Economic Activity*, 2021. 27

Romer, Christina D. and David H. Romer, "A Rehabilitation of Monetary Policy in the 1950's," *American Economic Review*, May 2002, *92* (2), 121–127. 6, 28

Sargent, Thomas, Noah Williams, and Tao Zha, "Shocks and Government Beliefs: The Rise and Fall of American Inflation," *American Economic Review*, September 2006, *96* (4), 1193–1224. 5, 32

Sims, Christopher A and Tao Zha, "Were There Regime Switches in U.S. Monetary Policy?," *American Economic Review*, February 2006, *96* (1), 54–81. 6

Staiger, Douglas, James H. Stock, and Mark W. Watson, "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives*, March 1997, *11* (1), 33–49. 14

Svensson, Lars E. O., "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules," *Journal of Economic Literature*, June 2003, *41* (2), 426–477. 3

_ **and Michael Woodford**, "Implementing Optimal Policy through Inflation-Forecast Targeting," in "The Inflation-Targeting Debate," University of Chicago Press, December 2004. 16

Taylor, John B., "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy*, 1993, *39*, 195–214. 6

Williams, Noah, "Adaptive Learning and Business Cycles," *Unpublished*, 2003. 5

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 2016, *48* (2-3), 253–291. 24

8 Appendix

A Long-run inflation expectations

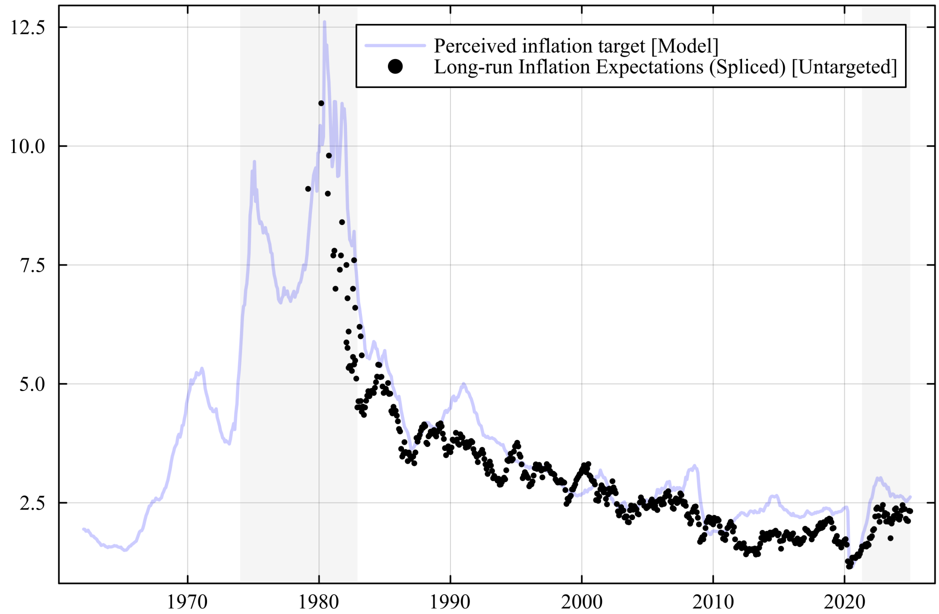
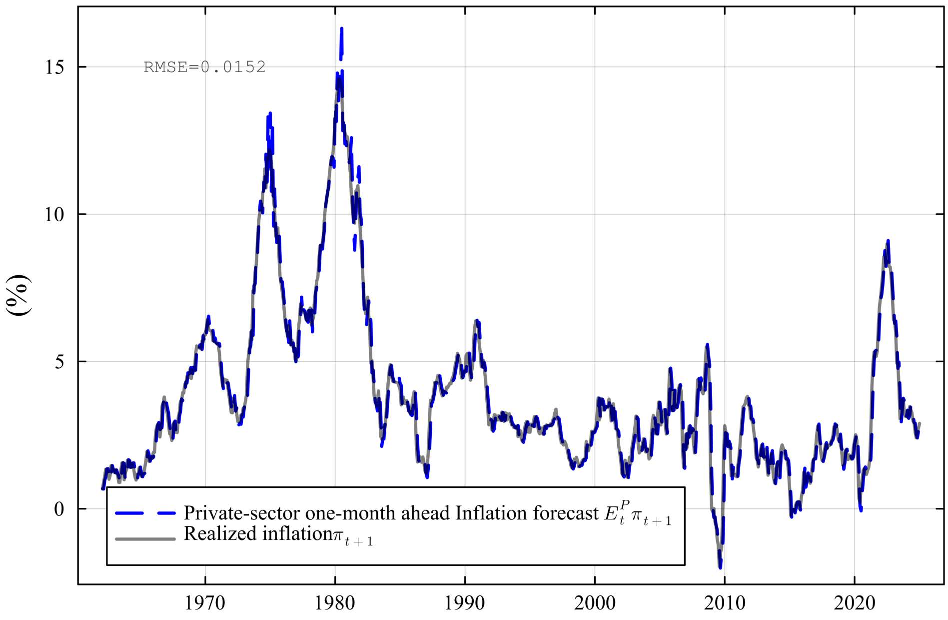


Figure 9: **Comparing with untargeted measures of long-run inflation expectations**

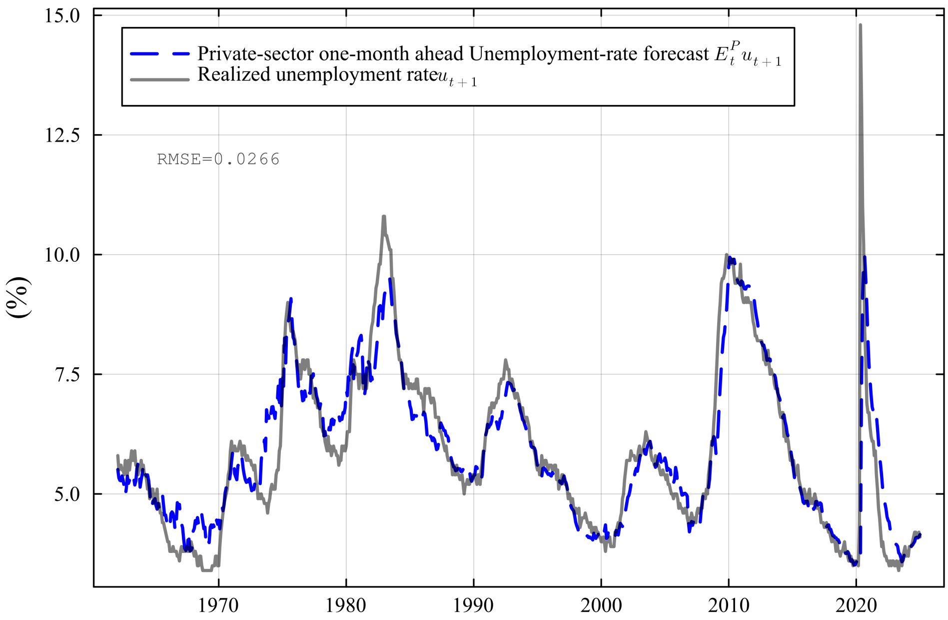
Note: Long-run Inflation Expectations series is constructed by splicing together 5-year ahead inflation expectations from Michigan Survey before 1982 and 10-year ahead inflation expectations from the Cleveland Fed afterwards (due to a shorter sample). The series is not used for estimation and is hence untargeted.

B Forecast performance of agents' model

If the learning algorithm arising from the agents' assumed model of economic dynamics implies forecasts that are persistently off-target, it wouldn't make sense to assume that the agents would continue using it. Rational agents would abandon the model. Hence, it is useful to check how well the agents' ex-ante forecasts agree with the realizations of the inflation, unemployment rate and interest rates or in other words if the agents' assumed model remains plausible given what they observe. Figure 11 plots the agents' ex-ante expectation of the policy rate (prior to announcement) and the realized effective Fed funds rate. Similarly, in Figure 10 I plot the agents' one-period ahead forecasts of inflation and unemployment rate against their corresponding realized values.



(a) Agents' one-month ahead forecast of inflation vs realizations



(b) Agents' one-month ahead forecast of unemployment rate vs realizations

Figure 10: **Forecast performance of agents' model**

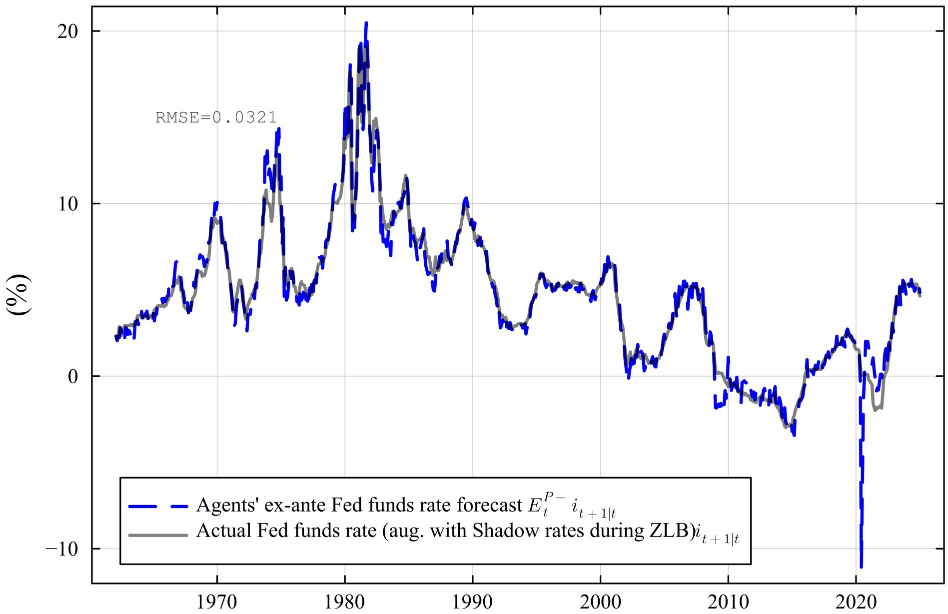


Figure 11: **How well do private agents track policy behavior?**

Note: The figure compares the private agents' expected nominal interest rates based on their ex-ante beliefs with the actual Fed funds rate (augmented with the shadow rate).

The learning model generates forecasts errors of small magnitude (in the RMSE sense). Thus, their model of the economy "works" - they wouldn't find a compelling reason to abandon learning. It is also worth noting that their expectations display a similar degree of persistence as the corresponding macroeconomic variables themselves. They are able to "match" the persistent behavior within a fully forward-looking model. Hence, forces such as price-indexation and habit-formation in consumption do not seem to be necessary to generate persistence in the macroeconomic aggregates once agents' real-time learning is taken into account, as also demonstrated by Milani (2007) and Erceg and Levin (2003).

C Filter Initialization

Table 2: Initialization and Burn-in

Parameter	Description	Initial value	Post burn-in
$E_{t-1}\pi_{t+1}$	Inflation expectations	2.00	0.70
$E_{t-1}x_{t+1}$	Unemployment-gap expectations	1.00	0.88
κ	Beliefs about Slope of PC	0.15	0.01
ξ_t^p	Beliefs about the persistent supply shock	0.00	-0.024
u_t^n	Beliefs about the natural rate of unemployment	5.00	6.36
r_t^n	Beliefs about the natural rate of interest	0.00	1.519
π_t^{*P}	Beliefs about the Fed's inflation target	3.0	1.59
λ_t^P	Beliefs about the Fed's weight on unemployment-gap stabilization	1.00	0.0098

Note: Initial values are set arbitrarily to plausible values and the filter is run for 48 periods, so that the beliefs are adjusted based on data. The post burn-in values indicate the agents' beliefs corresponding to 1962:01, which acts as the beginning of the effective sample period.

C.1 Initial covariance matrices for filtering

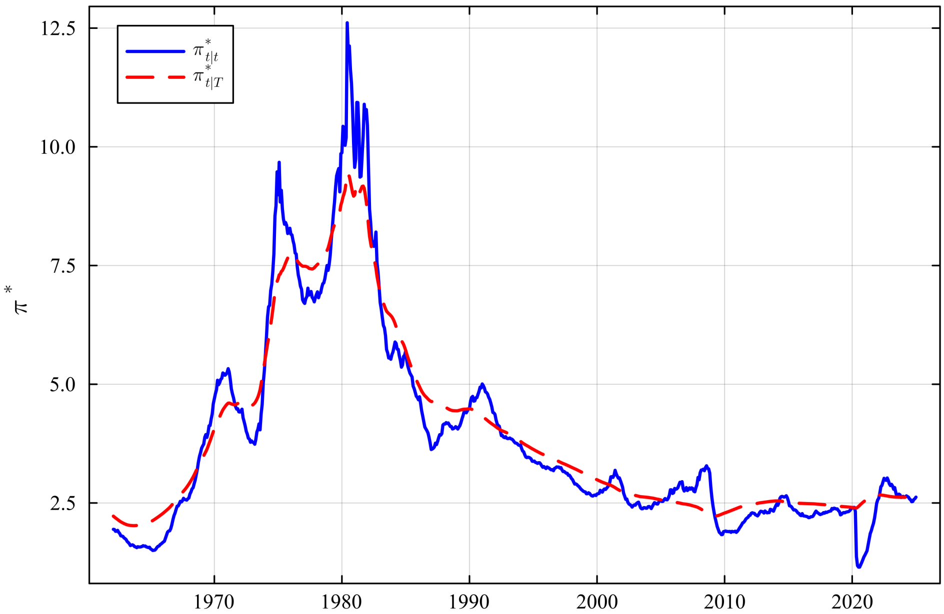
$$\text{For } \begin{pmatrix} \hat{\kappa}_t \\ \xi_t^p \end{pmatrix} : \quad P^{init} = \begin{pmatrix} 0.50 & 0 \\ 0 & 1.00 \end{pmatrix} \quad P^{post} = \begin{pmatrix} 0.449 & 0.0125 \\ 0.0125 & 1.765 \end{pmatrix}$$

$$\text{For } \begin{pmatrix} \log \pi_t^{*P} \\ \log \lambda_t^P \end{pmatrix} : \quad P^{init} = \begin{pmatrix} 100.0 & 0 \\ 0 & 100.0 \end{pmatrix} \quad P^{post} = \begin{pmatrix} 0.012 & 0.0038 \\ 0.0038 & 0.0134 \end{pmatrix}$$

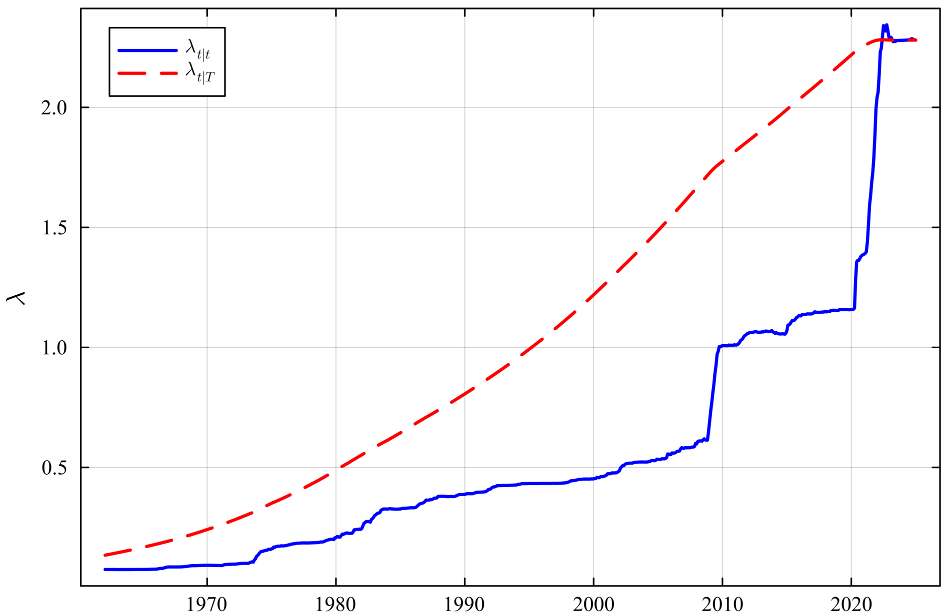
P^{init} indicates the initial value of the covariance matrix set at the beginning of the filter. P^{post} indicates the covariance matrix after the burn-in period.

D What were the Fed's "true" policy preferences?

While the parameters representing the "true" policy preferences of the Fed over the post-war period remain latent (it evolves as an exogenous structural variable as far as the model is concerned), it is possible to arrive at a best-guess using the model by obtaining the smoothed (backward-looking) estimates of the time-t policy preferences (Figure 12). These estimates represent the evolution of Fed's policy preferences in the post-war period from the perspective of a private agent in the model looking back at the end of the sample period (Dec 2024).



(a) π_t^*: Filtered *(solid)* vs Smoothed *(dashed)* estimates



(b) λ_t: Filtered *(solid)* vs Smoothed *(dashed)* estimates

Figure 12: **The Fed's "true" policy preferences**

Note: The figure plots the smoothed estimates of π_t^* and λ_t along with the filtered estimates. The smoothed estimates represent the agents' best-estimates of the parameters representing the Fed's policy preferences conditional on the entire sample period.

The estimates suggest that the Fed's true inflation target was upwards of 8 percent during the peak of the 80s-era inflation. Notably, the estimates of the time-varying inflation target are also in line with Ireland (2007) who uses a specification that is very different from the one used here.

Taken at face value, the smoothed estimates of λ suggest that Fed was hardly concerned with keeping the unemployment-gap closed during the 70s-80s. Over time however, the Fed has steadily emphasized closing the gap as part of its monetary policy strategy: which brings us to present-day. The estimated weight on unemployment-stabilization is almost twice as large as that on inflation-stabilization. The backward-looking estimates of λ_t display periods of departure from the real-time ones. This is because meaningful information regarding λ only arrives during periods where agents perceive large cost-push shocks driving policy tradeoffs (since λ is only relevant to policy when such a trade-off exists). Thus from the private agents' perspective, though λ has been evolving continuously, information regarding λ only arrives in clumps during episodes involving large persistent cost-push shocks driving policy trade-offs. Not much learning regarding λ takes place during "normal" times.

E Unemployment gap

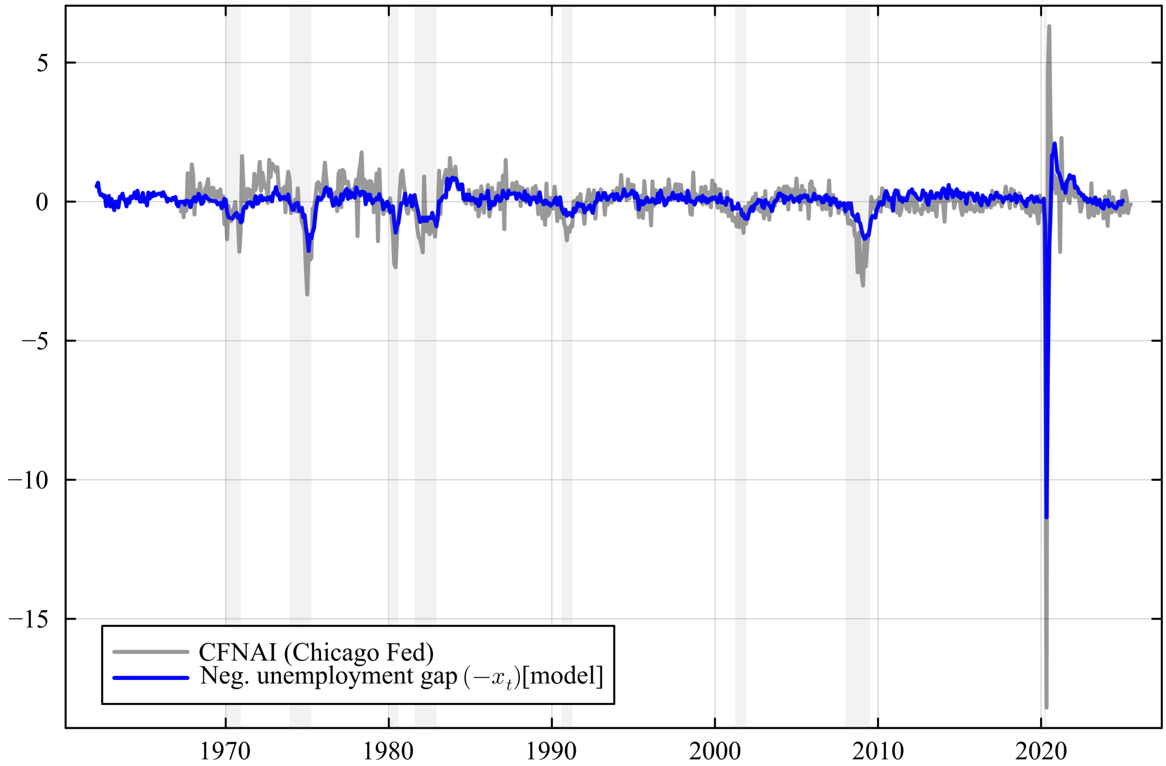


Figure 13: **Estimated unemployment-gap**

Note: The figure plots the agents' real-time estimates of the (negative of) unemployment gap with the CFNAI (Chicago Fed National Activity Index), a composite measure of real activity based on 81 series. The shaded regions indicate NBER recessions. CFNAI is obtained from the Chicago Fed website.

F Perceived natural rate of interest $r^n_{t|t}$

Figure 14 plots the evolution of the natural rate of interest as inferred by agents in real-time. This is a useful measure and serves as a barometer to gauge the stance of policy.

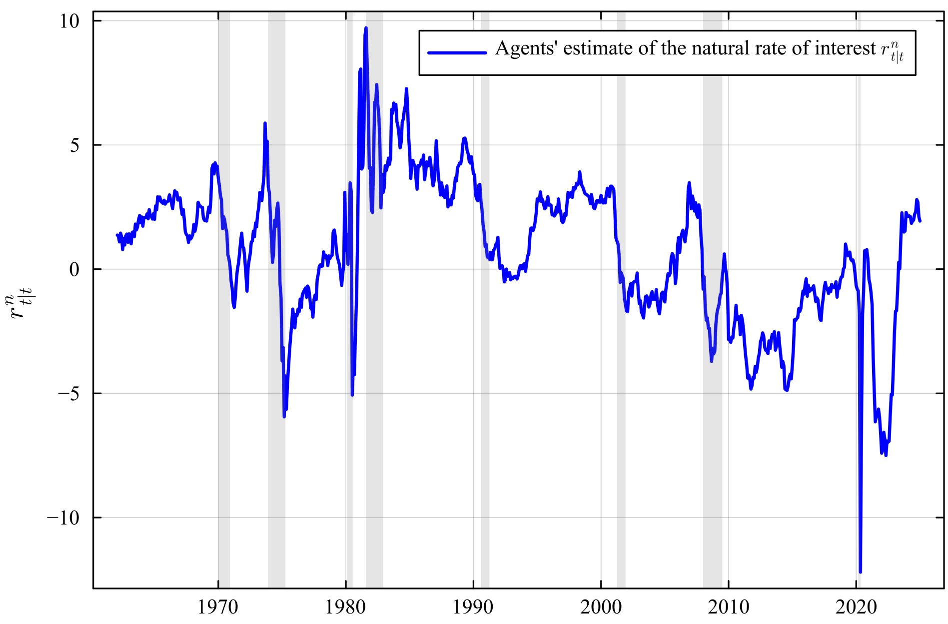


Figure 14: **Estimates of the natural rate of interest**

Note: Shaded regions indicate NBER recessions

The estimates are broadly in line with the secular decline reported in the literature. They typically fall with NBER recessions, which is rather intuitive based on the IS-curve.

G Has the "true" slope of the Phillips curve changed over time?

In the baseline model, we proceeded with the assumption of a time-invariant Phillips curve in order to keep the model parsimonious and avoid over-reliance on time-variation Phillips curve to explain the observations. However, one can statistically test if there has been a change in the true relationship between unemployment-gap and inflation during the pandemic (motivated by literature arguing that structural shifts in the Phillips curve during the pandemic period were an important factor driving the inflation surge).

In order to do this, we can construct an expectation-adjusted measure of inflation π_t^{adj} as:

$$\pi_t^{adj} = \pi_t - E_{t-1}^P \pi_{t+1}$$

This expectations-adjusted measure of inflation is regressed against the unemployment-gap variable x_t. The OLS result would not recover the true underlying κ since we would expect x_t to be correlated with the supply shock ξ_t. It will only be able to recover the reduced-form relationship between inflation and unemployment gap, not the structural one.

Table 3 shows that once expectations are taken into account, the relationship between inflation and the unemployment-gap is statistically significant - implying a 1% increase in the unemployment-gap associated with a roughly 10 bp decrease in inflation. It is worth noting that the estimate is quite close to the agents' beliefs about κ at the end of the sample shown in Figure 4 (≈ -0.11). However, as the second column shows, the dummy representing the change in slope post 2020 is not found to be statistically significant at the 5% level. The regression does not give convincing evidence of a change in the reduced-form relationship during the recent surge.

Table 3: Expectations-augmented Phillips curve: Examining change in slope

	Model 1 Baseline	Model 2 Change in slope post-2020
$\hat{x}_t$	**-0.106****	**-0.193****
	(0.037)	(0.063)
$\hat{x}_t \times \mathbf{1}_{\{t>2020\}}$	—	0.119
		(0.067)
Observations	756	756
R^2 (uncentered)	0.023	0.028
F-stat	8.322	10.42
p-value	0.004	3.44e-05

Notes: Dependent variable is π_t^{adj}. Model 2 augments Model 1 with the interaction $x_t \times \mathbf{1}_{\{t>2020\}}$, where $\mathbf{1}_{\{t>2020\}}$ equals one for dates from 2020 onward. HAC (Newey–West) robust standard errors (Bartlett kernel, maxlags = 6). R^2 is computed without centering because the model has no constant. Significance: * indicates that the coefficient is significant at the 5% level.

Another way to shed light on whether time-variation in the Phillips curve slope is an important feature of economic dynamics necessary to explain the data is to directly test for time-variation in the κ_t against the null of a constant-κ_t in the relationship:

$$\pi_t^{adj} = -\kappa_t x_t + \xi_t$$

If the data supports time-variation in κ_t, then relaxing the restriction of a constant κ should result in a significantly better fit, as measured by log-likelihood. A likelihood-ratio (LR) test is conducted, testing the alternate hypothesis of time-variation in κ against the null of a constant κ. The LR test is not statistically significant with $p \approx 1.00$. This suggests that the model finds no evidence in favor of time-variation slope of the Phillips curve once expectations are accounted for. The results of the test are reported in Table 4. Re-estimating the model with a time-varying slope κ (allowing for innovations with a reasonably small variance 10^{-3} over time) produces nearly identical paths of beliefs as the baseline. Overall, the results suggest that changes in the slope of the structural Phillips curve are not a necessary feature to explain post-war U.S. economic dynamics.

Table 4: Results of the Likelihood Ratio Test

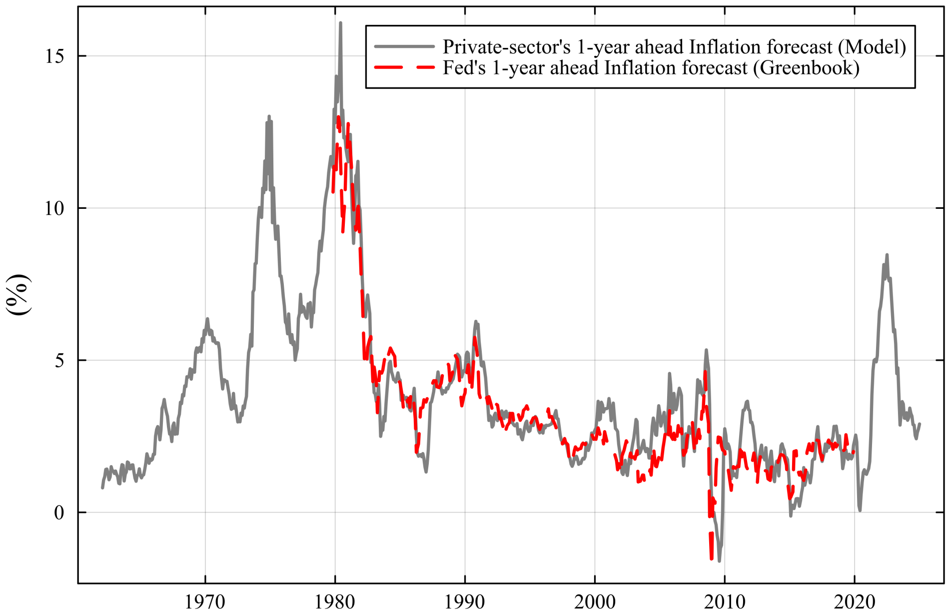
Model	**Log-Likelihood**	**Parameters**	χ^2	**df**	**p-value**
Restricted (Constant κ)	-402.460	3	3.1×10^{-7}	1	1.000
Full (Time-Varying κ)	-402.460	4			

Note: The restricted model assumes a constant κ coefficient, while the full model allows it to vary over time. The constant parameter case is nested within the full time-varying parameter model. The Likelihood Ratio Test (LRT) statistic (χ^2) is calculated as $2 \times (\log L_{\text{Full}} - \log L_{\text{Restricted}})$, with degrees of freedom (df) equal to the difference in the number of parameters between the two models.

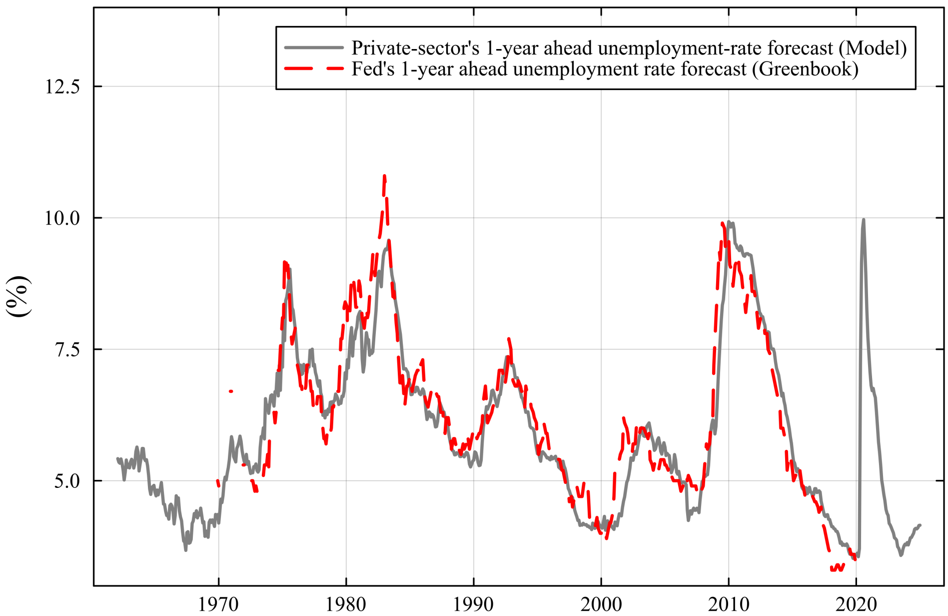
Overall, these results favor the time-invariant view of the Phillips curve adopted by the private agents in the model. Since the modeling framework in this paper explicitly accounts for endogenous formation of expectations, κ should not be interpreted as a purely a statistical relationship between inflation and unemployment described by Phillips (1958).

H Is the degree of information asymmetry small or large?

The analysis in this paper assumes very little information asymmetry between the Fed and the private-sector: (i) their beliefs about the structural parameters and shocks in the economy are shaped by the same data and the same economic model (ii) the filtering problem assumes that private-agents are able to track the true policy preferences efficiently, meaning that their tracked estimates never stray far away from the "true" underlying parameters describing the Fed policymakers' stabilization preferences. One way to test whether these assumptions hold well during the sample period is to compare the agents' forecasts as implied by the model with those from the Greenbook: the Fed's internal forecasts prepared by the staff before each FOMC meeting. If the Fed policymakers do hold significantly different beliefs compared to the private agents, then it would be reflected as a discrepancy in their forecasts. From Figure 15, we observe that Fed's internal forecasts as seen in the Greenbook seem to be in line with those of the private agents in the model. This suggests that any information advantage that the Fed possessed was of small order, at least to the extent that it influenced their outlook.



(a) **Inflation forecasts:** Comparing the Fed's 1-year ahead Headline CPI inflation forecast from Greenbook with 1-year ahead private-agents' forecasts from the baseline model



(b) **Unemployment rate forecasts:** Comparing the Fed's 1-year ahead unemployment rate forecast from Greenbook with 1-year ahead private agents' forecasts from the baseline model

Figure 15: **Macroeconomic outlook: Fed (Greenbook) vs Private Agents (Model)**