Monetary Policy as Decentralized Control: Near-optimality of Quantized Policy Rules

Shreeyesh Menon
Preliminary Draft

August 2025

Abstract

This paper studies macroeconomic stabilization as a cooperative game: the central bank and private agents share the common goal of stabilizing the output around potential but act under different information. Policymakers observe private signals about changes in the natural rate of interest that households and firms either lack or find costly to monitor; however, interest rate adjustments are costly making it infeasible to offset them completely. Private agents adjust inflation expectations based on noisy observations on the *output gap*. Importantly, central bank policy influences private sector's information by altering aggregate demand and serves as an implicit communication channel, informing them about fundamental shocks allowing for indirect stabilization through adjustment of inflation expectations. This paper draws an analogy of the problem to *Witsenhausen's counterexample*, a canonical problem in decentralized control theory, where optimal strategies are inherently non-linear. The analysis shows that quantized policy rules—such as adjusting interest rates in discrete steps—can outperform linear feedback rules like the Taylor rule.

1 Introduction

The stabilization role of monetary policy is inherently a problem of decentralized control. Policymakers determine the short-term nominal interest rate, while the evolution of the economy depends critically on how private agents form expectations about the future. Hence, optimal stabilization policy is a *team decision problem* in which multiple actors influence aggregate outcomes under possibly distinct information sets.

This paper studies optimal policy in a setting where the central bank and the private sector jointly aim to stabilize the economy against shocks, but under a *nonclassical information structure*. The central bank observes an economic disturbance—such as a change in the natural rate of interest—that is not directly observed by private agents. Policy adjustments are costly, reflecting quadratic costs for changes in the policy rate. Policy actions influence aggregate demand, which private agents only observe through a noisy channel. Agents can adjust expectations costlessly but do so under imperfect information: they are *blurry*.

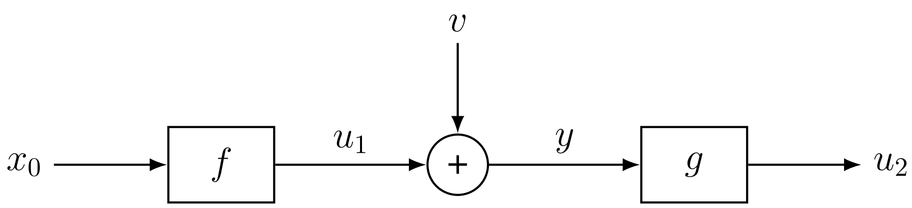
The key insight is that monetary policy serves a dual purpose: it both stabilizes the economy and communicates information about the underlying disturbance. In this sense, the central bank's policy choice acts as a *signal* to the private sector. The policy instrument therefore plays an informational as well as a stabilization role. The optimal policy must balance the incentive to smooth shocks against the value of conveying information to the private agents, who condition their expectations on the observed aggregate state.

This structure mirrors the logic of *Witsenhausen's counterexample* (Witsenhausen (1968)), a canonical problem in decentralized stochastic control. In Witsenhausen's setup, two controllers act sequentially to minimize a quadratic cost, but the second controller has imperfect information about the state, depending only on a noisy observation of the first controller's action. Linear strategies are strictly suboptimal: the first controller optimally chooses a *quantized* or signaling action that sacrifices immediate control to improve the information available to the second controller so they can stabilize costlessly. Analogously, in monetary policy, the central bank may deviate from the purely linear reaction rule to optimally transmit information to private agents through its policy choices.

The problem highlights that linear policy rules may not be optimal in settings where the policymakers' and private agents' information sets are different.

2 Witsenhausen's Counterexample

2.1 Setup



Consider a two-stage team problem with a nonclassical information structure. Let the initial state be a random variable $x_0 \sim \mathcal{N}(0, \sigma_x^2)$. There are two controllers:

1. Controller 1 observes x_0 and alters state to $u_1 = f_1(x_0)$ by adding a control input $u_1 - x_0$.
2. A noisy channel transmits u_1. Controller 2 observes $y = u_1 + v$, where $v \sim \mathcal{N}(0, \sigma_v^2)$, and chooses $u_2 = g(y)$, adding an input $u_2 - u_1$.

The total cost is quadratic:

$$J = \mathbb{E}\big[k^2(u_1 - x_0)^2 + u_2^2\big], \tag{1}$$

where $k > 0$ penalizes the first control effort.

2.2 Key features

- **Decentralized information.** Controller 2 does not know x_0 directly, only the noisy signal y.
- **Signaling incentive.** Controller 1's action u_1 both changes the system state and communicates information about x_0 to Controller 2 through y.
- **Nonclassical information structure.** The decision of the first controller affects the information available to the second, breaking the separation between estimation and control.

Despite the linear-quadratic-Gaussian (LQG) setting, the optimal encoding $f_1(x_0)$ is *nonlinear*. Witsenhausen (1968) showed that certain nonlinear "quantized" mappings outperform all linear strategies. The insight is that it can be optimal to use costly control actions for *communication*: small, discrete shifts in u_1 make y more informative to Controller 2 who can then stabilize by making cost-less adjustment $u_2 - u_1$.

3 A Stylized Economy

Consider a simple 2-period economy in which inflation π_t obeys a Phillips curve:

$$\pi = \pi^e + \kappa x \tag{2}$$

where π^e denotes private-sector inflation expectations, x denotes the output gap and κ denotes the slope of the Phillips curve (defining the mapping from aggregate demand to aggregate inflation).

For now, to keep the analysis elementary, we consider the output gap x to be given by the following IS curve:

$$x = -\sigma^{-1}(i - r^n - \pi^e) \tag{3}$$

where σ^{-1} is the intertemporal elasticity of substitution, i is the nominal rate of interest set by the central bank and r^n is the natural rate of interest.

Let CB denote the central bank and P denote the private agents (or firms). Initially, the economy is in steady state with natural rate of interest $r^n = \bar{r}$, nominal interest rate $i = \bar{i}$ and private sector inflation expectations $\pi^e = \bar{\pi}$. The output gap is zero.

1. At $t = 0$, there is a shock to r^n, causing output gap x_0
2. The central bank (CB) observes r^n and adjusts the policy rate to i
3. Intermediate output gap x_1 realizes
4. The private sector (P) observes y, a noisy realization of x_1 and forms expectations π^e
5. Output gap x and inflation π realize.

Together, both players together seek to minimize a loss function that penalizes deviations of output gap from zero and also has a quadratic penalty for interest rate adjustments.

This problem is equivalent to the problem described in the previous subsection.

3.1 Information structure

CB observes r^n before setting $i = f(r^n)$ and is hence equivalently choosing the intermediate output gap x_1, given by:

$$x_1 = -\sigma^{-1}(i - r^n - \bar{\pi}) \tag{4}$$

P observes the noisy signal y, given by:

$$\xi = x_1 + e, \quad e \sim N(0, \sigma_e^2) \tag{5}$$

Conditional on ξ, P chooses $\pi^e = g(\xi)$

Finally, output gap x realizes

$$x = -\sigma^{-1}(i - r^n - \pi^e) \tag{6}$$

The social loss function is given by:

$$L = x^2 + \lambda(i - \bar{i})^2 \tag{7}$$

where the first term imposes a quadratic penalty on output gap second term penalizes changes in the nominal interest rates

4 Discussion

The monetary policy problem illustrates the same conceptual difficulty as Witsenhausen's counterexample:

1. Control and estimation cannot be separated because the policy action shapes private information.
2. The optimal signaling policy may be nonlinear even in a linear-quadratic environment.
3. Information design and stabilization policy are intertwined—the central bank's communication (via i) is endogenous.

Both settings exemplify a *decentralized stochastic control* problem with nonclassical information, where signaling incentives create a rich interaction between optimal control and communication.

References

Witsenhausen, H. S., "A Counterexample in Stochastic Optimum Control," *SIAM Journal on Control*, 1968, *6* (1), 131–147. 2, 3